ARISTOTLE'S THEORY OF THE SYLLOGISM

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GÜNTHER PATZIG

ARISTOTLE'S THEORY OF THE SYLLOGISM

A LOGICO-PHILOLOGICAL STUDY OF BOOK *A* OF THE *PRIOR ANALYTICS*

Translated from the German by Jonathan Barnes

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DIE ARISTOTELISCHE SYLLOGISTIK

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TO THE MEMORY OF

JAN ŁUKASIEWICZ

(1878–1956)

TRANSLATOR'S NOTE

I thank Professor J. L. Ackrill for his constant interest, Martin Holt for a felicitous suggestion, and my wife, whose work as much as mine this translation is.

J.B.

PREFACE TO THE ENGLISH EDITION

The present book is the English version of a monograph 'Die aristotelische Syllogistik', which first appeared ten years ago in the series of *Abhandlungen* edited by the Academy of Sciences in Göttingen.[1]

In the preface to the English edition, I would first like to express my indebtedness to Mr. J. Barnes, now fellow of Oriel College, Oxford. He not only translated what must have been a difficult text with exemplary precision and ingenuity, but followed critically every argument and checked every reference. While translating it, he has improved the book. Of those changes which I have made on Mr. Barnes' suggestion I note only the more important ones on pages 4, 12, 24sq, 32, 39, 61sq, and 158.

Since the second edition of the German text appeared in 1963 some further reviews have been published, or come to my notice, which I have been able to make use of in improving the text of this new edition.[2] I must mention here especially the detailed critical discussions of my results and arguments published by Professor W. Wieland in the *Philosophische Rundschau* **14** (1966), 1–27 and by Professor E. Scheibe in *Gnomon* **39** (1967), 454–64. Both scholars, while agreeing with the main drift and method of my interpretation, criticise some of my results and disagree with some of my arguments. It would not be possible to discuss these technical matters here with the necessary thoroughness. I shall do this elsewhere and refer the reader in the meantime to these reviews which deserve close study. Especially important are, I think, Scheibe's new explanation of the exclusion of singular terms in *A* 1–7 (l.c. 457) and Wieland's new account of the proofs by ecthesis (l.c. p. 25). Both reviewers have also, by private communication, given me useful hints for corrections and additions, which I gratefully acknowledge. One point of importance, however, raised by Wieland in his review, should not be passed over in silence: Wieland is generally less ready than I to suppose that Aristotle "knew" some law of logic which he makes use of in his reasoning. I agree with Wieland that in some cases I have been somewhat overconfident. However, the matter is probably less clear-cut than he assumes.

"Using" a law of logic and "knowing" it are not, it seems to me, related as "use" and "mention" of names are. Between mere use and clear knowledge of a law there may be some intermediate state – we may be half-aware of it. One may reason according to some law of logic without realising that there is such a law or that it is a logical law. One may also be sure one's reasoning is a matter of logic, without being able to formulate the law or give an account of its validity. So, while accepting Wieland's maxim (l.c. p. 2 sq., 22 sqq.), I suggest that its practical application is difficult and must leave room for debate in most cases.

In appendix A the reader will find, as a supplement, the text of a short article that appeared in *Mind* **68**, 1959. I thank the editor of *Mind*, Professor G. Ryle, for his kind permission to reprint it here. I also wish to express my thanks to Dr. Dorothea Frede for her assistance in the proof-reading of the translation.

That this book is now available in an English edition must be especially welcome to the author in view of the fact that the two most important works on its subject matter to appear in our time, Sir David Ross' commentary (1949) and the monograph of Jan Łukasiewicz (1951), were published in English too. It will be evident to the reader how much the author has profited from the study of both these works, however much he disagrees on occasion with both authors. Perhaps it is too much to hope that this book will deserve a place beside these two classics. It will be enough if it helps some readers to understand Aristotle's logical texts better and if it contributes to the recognition of the fundamental fact – so often neglected – that the history of philosophy requires a judicious balance between hermeneutic methods and skill in dealing with the philosophical problems which the texts are trying to settle. Since Łukasiewicz has been my example in trying to reach this balance since I began my studies in the history of logic, I wish to dedicate the English version of my book to the memory of this great Polish logician.

G. PATZIG

Göttingen, October 1968

NOTES

1. A third edition of the German text and a translation into Rumanian, produced under the auspices of the Rumanian Academy of Sciences in Bucharest, are to appear at the same time as this translation.
2. P. S. Popov, *Voprosy filosofii*, 1961, 178–179;
   D. Campanale, *Rass. Sci. filos.* **14** (1961), 364–365;
   Luciano Montoneri, *Sophia* **30**, 3–4, July-Dec. 1962;
   K. Ennen, S.J., *Scholastik* **40** (1965), 408–11;
   Czeslaw Lejewski, *Journal of Symbolic Logic* **31** (1966), 103–104;
   W. Wieland, *Philosophische Rundschau* **14** (1966), 1–27; and
   E. Scheibe, *Gnomon* **39** (1967), 454–464.

CONTENTS

FROM THE PREFACE TO THE FIRST EDITION

Renascent sciences reward their forebears with indifference or hostility. Later, however, historical curiosity grows, and present attainments must be measured against the accomplishments of the past: knowledge and self-knowledge alike are deepened by comparison with earlier incarnations. Sometimes forgotten achievements are rediscovered and their fertility revealed: and they are restored to their rightful place in history.

The massive advance in formal logic since 1879, the year in which Gottlob Frege's *Begriffsschrift* appeared, has drawn in its wake a revived interest in the history of logic. The expansion of logic over whole new domains – the logic of propositions, of relations, etc. – the more stringent demands for rigour in proof and conceptual precision, and the mechanical aids of the calculi, have long since demonstrated their importance in the technical manipulation of logical problems. In the last decades they have been put to serve the interpretation of logical texts of earlier centuries; how rich the yield here too may be was first shown by Heinrich Scholz' *Geschichte der Logik* of 1931 and by Jan Łukasiewicz' paper 'Zur Geschichte der Aussagenlogik' of 1935.

Formal logic is perhaps unique among the sciences in being able to trace its origins back to one classical work – Aristotle's *Analytics*. A contemporary presentation of Aristotle's logic was therefore devoutly to be wished: it was provided by Łukasiewicz in his book *Aristotle's Syllogistic from the standpoint of modern formal logic* (Oxford, 1951). This book, it is no exaggeration to say, heralded a new era in the history of Aristotelian exegesis, a history spanning two thousand years. Łukasiewicz proved that Aristotle's syllogistic differs fundamentally from the so-called classical syllogistic, which travelled by devious paths through Boëthius, Petrus Hispanus and other writers from its origins in late antiquity to its present place in our text-books. He found further in Aristotle's text many ideas explicitly formulated or at least intimated which are closely related to theorems of modern mathematical logic. He showed, in a word, that as a logician, even "from the standpoint of

modern formal logic", Aristotle fully merited the admiration which posterity had so long bestowed on him for the wrong reasons.

The vigour with which Łukasiewicz tried to relate Aristotle's text in its entirety to the developed system of modern mathematical logic, and to construe Aristotle's syllogistic as an embryonic form of modern logic, stimulated and tautened scholarly discussion of the *Analytics*: however, these two designs are, from another point of view, the basic faults of a book which otherwise shows such excellence and such uncommon acumen. If a concrete text, transmitted to us from the past, is treated as a mile-stone on the way to some other text, as an adumbration of later knowledge, then justice will not be done to the text itself or to its author. The published objections which have so far been raised against Łukasiewicz' method of interpretation all point in this direction – if they are not based on simple ignorance of or antipathy toward mathematical logic. On the other hand, Łukasiewicz' book has shown incontrovertibly that a mere philological interpretation, innocent of detailed logical expertise, is not sufficient. This is corroborated by almost every page of W. D. Ross' invaluable edition of the *Analytics* (Oxford, 1949) – a book which Łukasiewicz was not able to consult.

In these circumstances it seemed worth while to try to interpret the Greek text of the *Prior Analytics* in a way that allows Aristotle to speak for himself, to pose his own problems and to answer them in his own manner. The long tradition of classical interpretation had to lend us the sensibility with which it has listened to the language of the text; only so could the Aristotelian half of Aristotle's logic be grasped in its purity. To make the logical half apparent, the aid of modern logic and the results of Łukasiewicz' exertions were indispensable. Thus, to put it picturesquely, we can say that the present investigations are an attempt to bridge the gap between Łukasiewicz' logical systematization of the text and the traditional interpretations of the classical scholars. The sub-title of the work, 'a logico-philological investigation', is meant to indicate just this conjunction.

The following five chapters, devoted to individual problems, try to prove that such a method of interpretation can lead to a new and more exact understanding of the text.

G. PATZIG

Göttingen, February 1959

FROM THE PREFACE TO THE SECOND EDITION*

The friendly reception which this book has met with in the scholarly world[1], and the fact that it has for some time been out of print, have led me to hope that my effort to unite the logician's systematization of Aristotle's syllogistic with the classical scholar's textual analysis of the *Prior Analytics* has provided some useful starting-points towards the interpretation of Aristotle's logic.

. . . .

I have received most valuable help in the revision of this book from the instructive criticisms and developments of my views proposed by J. L. Ackrill in his review of the first edition. I am especially grateful to M. Bierich, J. Mau and C. F. von Weizsäcker for valuable comments by letter and in conversation. Numerous important improvements have been contributed by my assistant R. Drieschner; he has also helped in extending the bibliography and in preparing the new impression for the printers.

The principal results of chapters III (on the 'perfection' of syllogisms), IV (on the definition of the figures), and V (on Aristotle's methods of proof and rejection) have met with a pleasing degree of acceptance. ... With regard to chapter I (on the difference between the Aristotelian and the traditional syllogism), I have come to see that the question whether Aristotle's logic is a logic of propositions or of rules cannot be settled in favour of propositions as confidently as, following Łukasiewicz, I had formerly thought.

It seems likely that this way of putting the question is improper. It has been shown that the presentation of Aristotle's syllogistic with the help of Russell's 'material implication' (Quine's 'conditional'; Lorenzen's 'subjunction') and the logical quantifiers, (x) and $(\exists x)$, allows most but not all of the peculiarities of Aristotle's system to be represented. Of the formal devices developed in the literature on mathematical logic, Lorenzen's 'logical implication'[2] now seems to me to come nearest to Aristotle's "If... then..." connective[3].

. . . .

The fundamental criticism which, in chapter II, I thought had to be made against Aristotle's distinction between 'relative' and 'absolute' necessity now seems to me to overshoot the mark[4]. It remains true that many of Aristotle's formulations are open to misconstruction and are potentially misleading, where it is a question of distinguishing between the necessary *truth* of the *implication* between premisses and conclusion and the necessity of the *fact* which the conclusion states. But a deeper study of the text shows that Aristotle did see very clearly the real difference between these two sorts of necessity. And that is of course the most important point.

. . . .

If I were to rewrite this book, I should be still more circumspect than I was before in ascribing logical errors to Aristotle. We may not assume that he can have made no mistakes at all; but it seems to me reasonable that we should first see if we have not misunderstood Aristotle before we decide to reproach him. [Secondly, as I have indicated, I have become a little more sceptical of the sesame of modern formal methods in historical enquiries in the field of logic; but I believe that we are no longer at liberty to choose any other method.]

Secondly, in some places my criticisms of nineteenth and twentieth century scholars now seem to me much too harsh in substance and in tone. A new point of view carries with it the danger of onesidedness; I have come to see that even the views – which I still reject – of Waitz, Trendelenburg and Maier are backed by reasons which must be taken seriously. In the work of Prantl alone I can still find no such reasons.

Finally, I would like to take this opportunity to remove a misunderstanding which some of my statements might suggest, and which I find most clearly expressed in Allan's review (l.c. p. 36): my criticism of the nineteenth century interpreters who based Aristotle's syllogistic on his metaphysics may easily be taken to imply that these two pillars of philosophy stand completely unrelated in Aristotle's works. I never intended to advance this view, which is, historically speaking, evidently false; it is quite different from the thesis which I believe I have established: that the *validity* of the propositions in Aristotle's syllogistic can, neither in fact nor in Aristotle's opinion, be thought dependent on the *truth* of certain ontological propositions. It is consistent with this view both that Aris-

totle's *presentation* of his syllogistic is unconsciously influenced in many ways by his ontological predilections, and also that the marrow of Aristotle's ontology contains views which mirror his logical tenets. If a causal connexion between Aristotle's logic and his ontology must be found, it seems to me more correct to base his ontology on his logic than the other way about. This relationship appears to recur in Leibniz.

G. PATZIG

Hamburg, February 1963

NOTES

* Certain of the detailed points made in this preface have been transferred in this edition to the body of the text.

1. The detailed reviews which have come to my notice are those by A. Lumpe (*Philos. Lit. Anz.* **13** (1960), 147–149), by J. Mau (*DLZ* **62** (1961), 112–115), by D. J. Allan (*Class. Rev.* [N.S.] **11** (1961), 34–36), and by J. L. Ackrill ('Critical Notice', *Mind* **71** (1962), 107–117). The fact that these reviewers, all classical scholars by trade, have understood my argument perfectly, even if they have not accepted it all, is particularly gratifying in view of W. Theiler's prophecy (*Mus. Helv.* **18** (1961), 240) that only a few classicists would "take off into the abstract air of mathematical logic" and the majority "would prefer to remain on *terra firma* with Maier, ... Hartmann and Solmsen, where Aristotle's logic can be seen as an offshoot of Plato's ontological speculations."
2. Cf. Lorenzen, *Formale Logik* (1958), §§ 2, 7, 8.
3. I am strengthened in this belief by K. Ebbinghaus' book, *Ein formales Modell der Syllogistik des Aristoteles*, Hypomnemata, Heft 9, Göttingen, 1964.
4. Thus I accept the criticisms on this point offered by Lumpe, Mau and Ackrill.

CHAPTER I

WHAT IS AN ARISTOTELIAN SYLLOGISM?

§ 1. The Traditional Form of the Syllogism

A celebrated argument at once springs to mind when the syllogism comes under discussion. The Scholastics called it *Barbara*, and it runs as follows:

All men are mortal,
Socrates is a man,
therefore: Socrates is mortal.

This sort of inference is generally represented with a line under the premisses in place of the "therefore", and with variables for concrete terms:

All M is P,
S is M
―――――
S is P.

These facts are familiar; we may further assume it well-known that this type of argument was first examined by Aristotle (384–322 B.C.) in his *Analytics*, and, perhaps, that such inferences occur in *four* 'figures' – although Aristotle, oddly enough, recognised only *three*. These figures are distinguished by the position of the middle term, usually symbolised by M, which alone of the three terms of a valid inference appears in both 'premisses' (the propositions *from which* we infer) but does not appear in the 'conclusion' (the proposition *to which* we infer). If the middle term stands, as it were, chiastically, as it does in our example, the syllogism is in either the first or the fourth figure of *traditional* logic, the first figure in *Aristotelian* logic; if it stands at the end of both premises, we have the second figure, if at the beginning of both, the third.[1]

The 'moods' of each 'figure' are differentiated from one another solely by the logical character of their premisses and conclusion, which must always be either universal or particular and either affirmative or negative propositions. The mediaeval logicians introduced the symbols *a*, *e*, *i*, and *o* to stand for these types of proposition: "*SaP*" is the customary ab-

breviation for "All S is P", "SeP" for "No S is P", "SiP" for "Some S is P" and "SoP" for "Some S is not P". *a*, *e*, *i*, and *o* together make up all the possible *logical* relations between two terms; moreover, since *o* and *i* are the *negations* of *a* and *e* respectively, *a*, *e*, and not-*a* and not-*e* (or *i*, *o*, not-*i* and not-*o*) would suffice. Since the possible ways of combining four elements into sets of three members number 4^3 or 64, there are for each figure 64 different groups of three propositions; of these only six groups in each case are valid *syllogisms*. Aristotle indeed names only four moods in the first and second figures; for if an inference validates a *universal* conclusion, he does not separately count the corresponding inferences to a *particular* conclusion which is of course also (*a fortiori*) valid.

The traditional names[2] of the valid moods (*Barbara*, *Ferio*, *Calemes*, *Fresison* etc.) were ingeniously contrived to reveal their formal properties: all the names are trisyllabic, the vowel of each syllable indicating the logical form of the proposition which it represents; thus *Ferio* stands for the syllogism $MeP \& SiM \rightarrow SoP$.[3] The names also serve to show how the moods can be 'reduced' to syllogisms of the first figure, and thereby be proved: this is connected with Aristotle's doctrine that syllogisms of the first figure, and they alone, are 'perfect'. (See Chapter III.) Again, from the evidence supplied by the names we can easily discover to which figure any inference whose scholastic name we know must belong; in practice, however, it is more convenient to get by heart the well-known mnemonic[4] which sets out the distribution of the moods in the figures. In the course of this enquiry I shall regularly note in brackets after the name of a syllogism the figure to which it belongs. With the help of the schemata of the figures on page 13 and the explanation of the names and their vowels given above, the reader will always find it perfectly easy to construct in its traditional form any syllogism to which I refer. *Darapti* (III), for example, could at once be written out as $MaP \& MaS \rightarrow SiP$.

Earlier writers[5] have occasionally pointed out that an Aristotelian syllogism differs markedly, from a formal point of view, from the inference-schema of traditional logic which we cited above; but the first to insist with due emphasis that these differences be taken into account was Łukasiewicz, who treated them comprehensively and in detail.[6] It is expedient to start with a brief discussion of these differences since the far-reaching misunderstandings of Aristotle's syllogistic theory by later com-

mentators, with which we shall have to concern ourselves in the course of this study, owe their origins in large part to the equation of the *Aristotelian* with the *traditional* syllogism.

§ 2. Propositions and Rules of Inference

A traditional syllogism is a compound of three propositions, which are usually written without connectives one below another. A line beneath the two premisses or a "therefore" before the third proposition indicates that the last 'follows from' the other two. The traditional syllogism is therefore not itself a proposition that can be true or false: it is a compound of three such propositions. The assertion that such a compound of propositions is a syllogism (i.e. a valid syllogism) is *ambiguous*, its meaning depending on whether the propositions are composed of concrete terms or of variables. Our first example was:

All men are mortal,
Socrates is a man,
therefore: Socrates is mortal;

the assertion that this is a valid syllogism means that the premisses are true and hence the conclusion too is true.[7] If, on the other hand, an inference contains only variables, as in our second example:

$$\frac{\begin{array}{l}\text{All } M \text{ is } P,\\ S \text{ is } M\end{array}}{S \text{ is } P.}$$

then to assert that the syllogism is valid means that, if terms are substituted for M, P, and S such as to make both premisses into true propositions, then the conclusion, "S is P", must also be recognised as true. From a logical point of view the first syllogism is a *proof of the conclusion*, the second a *rule* of proof. Traditional logic tends to ignore completely or even explicitly excludes the case where the premisses are *false*.

An Aristotelian inference is, by contrast, a proposition having the form of an "if ... then ..." implication, its antecedent is the conjunction of the premisses (i.e. the two premisses conjoined by an "and" into one proposition), and its consequent consists of the conclusion. To bring our initial example into Aristotelian form it would be necessary to write:

> *If* all men are mortal *and* Socrates is a man, *then* Socrates is mortal.

This is a *single* proposition, although one put together from several propositions, and as such it is immediately true or false – there is no need to seek further, extraneous information, to consider what is the significance of the order of the propositions, of the line under the premisses, and so on. In Aristotle syllogisms with variables in the place of concrete terms preponderate; indeed, in the systematic exposition of syllogistic (*APr. A* 1–7), with the interpretation of which we shall be primarily concerned, there is no example of a syllogism with concrete terms. Nevertheless, propositions of the form:

> If *A* belongs to no *B* and *B* to some *C*, it is necessary that *A* does not belong to some *C* (*APr. A* 4, 26a25–27),

are still *propositions*; they are true or false, and their truth or falsehood can be proved by the application of definite rules. This also explains how Aristotle can admit syllogisms with false premisses as perfectly valid.[8]

It is not, of course, correct to say that Aristotle never produced a syllogism couched in the traditional formulation, which was current in late antiquity, in which the premisses stand unconnected after one another and the conclusion is introduced by "therefore" (ἄρα).[9] Such forms do in fact sometimes occur in Aristotle, in places where interpolation is inconceivable. (E.g. *APr. A* 38, 49a32–35; *APst. A* 6, 75a9–11; *A* 13, 78b24–28.) The majority of these cases, however, concern arguments with concrete terms: it is natural to put the syllogism in the form of a proof if it is known on other grounds that the premisses are true. At all events, Łukasiewicz's assertion, invalid for the *Analytics* as a whole, does hold for the systematic treatment of the syllogism in the first seven chapters of the *Prior Analytics*.

§ 3. Singular Terms

In his syllogistic Aristotle uses no singular judgments ("Socrates is a man") as premisses of valid inferences. It is true that Łukasiewicz (AS, pp. 5–7) is wrong in saying that in the *Prior Analytics* Aristotle quotes *no* argument containing singular propositions; for in *B* 27 (70a16–20) a syllogism in *Darapti* (III) – albeit an invalid one – occurs in which "Pittakos" stands as the middle term; and "Aristomenes" and "Mikkalos"

appear in one passage (*A* 33, 47b15 sqq.) as terms of premisses, from which, admittedly – on other grounds –, nothing can be deduced. Nevertheless, it remains an important and indubitable fact that in his survey of the possible propositional forms, *a*, *e*, *i*, and *o* (*APr. A* 1, 24a16 sqq.), Aristotle fails to mention singular propositions, and he is obviously inclined to exclude them from his systematic discussions of syllogistic form.

On his reasons for doing this the best commentators are in agreement: Ross[10] draws attention to Aristotle's remark that discussions and enquiries of a scientific nature almost always concern objects which are neither individuals nor terms of the highest generality (categories) – this concludes a description of the three groups of ὄντα (sic) according to their predicability (*APr. A* 27, 43a25–43). Łukasiewicz, (AS, p. 6), referring to the same passage, thinks that Aristotle wished to bar from his syllogistic all those terms which could not appear in true propositions both in subject and in predicate position. For in each figure there must be a term which occurs once as subject and once as predicate: *M* in the first figure, *P* in the second, *S* in the third, (and all three terms in the fourth, which Aristotle did not discuss).

A peculiarity of the language of this passage may afford a somewhat deeper understanding: when Aristotle distinguishes 'proper names', 'species', and 'categories' according to their *predicability*, he speaks not of three kinds of ὅροι or terms but of three kinds of '*beings*'. Łukasiewicz (AS, p. 6) simply calls this an *error* on Aristotle's part: *things* cannot be predicated, only *terms* can[11]. That is obvious; however, we are dealing with Aristotle, and in such a case we ought perhaps to try to understand why he uses such peculiar language. There seems to be a noticeable tendency here to avoid the expressions "ὅρος", "ὄνομα", and "λόγος" for categories and individuals or for their names: chapter *A* 27 of the *Prior Analytics* does not offer a division of terms into individual-terms, species-terms and categories, but a division of 'beings' into individuals (sensibles – and their names), universals (and the terms standing for them), and the categories, which are not properly terms at all (they cannot be *defined*, since every definition must be per genus proximum et differentiam specificam, and in the case of the categories there is no longer a genus proximum). 'Individuals', according to Aristotle (cf. *APst. B* 19), are grasped by means of perception; the first principles, among which are the cate-

gories, are known by intuition (νοῦς); while *terms* (in the proper sense) are the province of ἐπιστήμη (scientific knowledge). Syllogistic, then, will be the theory of certain relations between terms of the middle class which are neither names of individuals nor categories.

If, as Ross has good grounds for conjecturing, Aristotle had really restricted the argument-range of his term-variables to this middle class *because* the sciences do as a matter of fact for the most part deal with such terms, the rather unsatisfactory consequence would follow that Aristotle *oriented* his logic by reference to the linguistic habits which the sciences *as a matter of fact* exhibit. Łukasiewicz realized the seriousness of this and he therefore adduced as a possible motive ("the true reason", AS, p. 6) for Aristotle's decision the further, *logical*, fact that in each of the three Aristotelian figures one term must always occur once as subject and once as predicate. Since singular terms cannot function as predicates, and categories (according to Aristotle's explicit, if disputable, doctrine) cannot function as subjects, Aristotle excluded these types of term from the range of the variables *P*, *M* and *S*. It should be pointed out, however, that there are terms in Aristotle's figures which occur only as subject (*S* in I and II) and others which occur only as predicate (*P* in I and III)[12]: and in these positions at least singular terms or categories could occur without causing qualms.

The question can, however, be given a more decisive answer, and I think, a satisfactory one. For the division of 'ὄντα' into the three classes of *A* 27 serves to prepare the ground[13] for the following chapter, *A* 28, in which Aristotle sets out procedures whereby certain propositions of the forms *SaP*, *SeP*, *SiP* and *SoP* can be *proved.* Here he assumes without further argument that for each term there exists both a term which may appear as subject to it (subordinate term) and a term which may appear as predicate of it ('superordinate' term). The recommended procedure for proving *SaP*, for example, consists in examining the set of predicates of *S* and the set of subjects of *P* for a common element. This element could then serve as the middle term in a syllogism:

$$MaP \,\&\, SaM \rightarrow SaP.$$

An element common to the subject-terms of *S* and the subject-terms of *P* permits the construction of a syllogism in *Darapti* (III):

$$MaP \,\&\, MaS \rightarrow SiP.$$

To prove a proposition of the form *SeP*, Aristotle introduces the notion of a set of terms 'contrary' to a given term (ἃ μὴ ἐνδέχεται αὐτῷ παρεῖναι, 44a4–5), and establishes that an element common to the set of contrary terms of the predicate term and the set of predicate terms of the subject term allows a syllogism in *Cesare* (II).[11] *SoP* may be proved by finding a common element among the contrary predicates of *P* and the subjects of *S*; the syllogism (after conversion of the major premiss, cf. n. 14) is in *Felapton* (III):

$$MeP \,\&\, MaS \rightarrow SoP.$$

The expression "one must examine the set of subject (predicate, contrary) terms of *S* (*P*)" clearly presupposes that in each case these sets have at least one member. The following *axioms* therefore hold for the procedures of *A* 28:

(1) Every term which can appear in a syllogism as the value of a variable *P* (*S*), has at least one proper *subordinate term*. I.e.: for all *P* (*S*) there is a term *M* such that *MaP* (*MaS*) but not *PaM* (*SaM*) holds.

(2) Every term which can appear in a syllogism as the value of a variable *P* (*S*), has at least one proper *superordinate term*. I.e.: for all *P* (*S*) there is a term *M* such that *PaM* (*SaM*) but not *MaP* (*MaS*) holds.

(3) Every term which can appear in a syllogism as the value of a variable *P* (*S*), has at least one *contrary term*. I.e.: for all *P* (*S*) there is a term *M* such that *MeP* (*MeS*) holds – so that *P* (*S*) can be predicated of no element of the class defined by *M*.

It is clear that singular terms do not satisfy axiom (1) and that categories (on Aristotle's definition) do not satisfy axiom (2). Furthermore, axioms (3) and (1) would exclude the *universal class*, since its contrary is the *null class* and this has no subordinate term below it. But the universal class has already been excluded by Aristotle on other considerations (*Met.* *B* 3, 998b22–27). It would be rash to assert that Aristotle realized clearly that the procedure of *A* 28 presupposes these three axioms. On the other hand, it is most striking that the argument of *A* 27 aims at excluding precisely those terms which do not satisfy our axioms. We may therefore propose with some confidence that the real motive for Aristotle's restricting, in *A* 27, the range of the syllogistic variables to terms of middle generality, was his desire to found the procedure described in *A* 28 on firm logical grounds. For the fact adduced by Łukasiewicz is not

once mentioned in our passage; and the reason alleged by Ross for the restriction – Aristotle's deference to the actual practice of the sciences, which he describes with care (σχεδόν, μάλιστα: 43a42sq) – is far better understood as a justification of his procedure, an assurance that it does not prejudice the sciences. Aristotle's syllogistic can now be defined, more precisely than was done on page 5sq., as the theory of certain relations between terms which satisfy the axioms (1)–(3). What these 'certain relations' are must now be made clear.[15]

§ 4. Formulation of the Premisses

An Aristotelian syllogism differs from a traditional syllogism (a) by its being a proposition, (b) by the rules governing the range of its variables, and (c) by the linguistic expression used for the logical relations in which the variables or their arguments stand to one another.

We have seen, (a), that an Aristotelian syllogism is a *single* proposition of the form "If *A*, then *B*", the antecedent being the conjunction of the premisses and the consequent the conclusion. A traditional syllogism, on the other hand, is a rule of inference, asserting that from two propositions of a certain kind we can pass to a third proposition. Thus the traditional syllogism is by its nature a proposition *about* three propositions, whereas the Aristotelian syllogism is a proposition *constructed out of* three propositions.

We saw, (b), that only terms which satisfy the axioms (1)–(3) of page 7 are admitted as arguments for the variables of an Aristotelian syllogism. This restriction does not apply to the *traditional* syllogism.

Compared to these important *logical* differences, the contrasting *expressions* used by Aristotle and traditional logic to stand for the logical relations between the variables represent a difference which is merely *terminological*. However, this difference, precisely because it has been generally disregarded, has played an important part in the traditional interpretation of Aristotle's syllogistic. It is largely on account of this that Aristotle's distinction between 'perfect' and 'imperfect' syllogisms, and hence his preference for the first figure, have been often discussed and never understood. Again, unless this terminological difference is noted, Aristotle's definitions of the terms (major, middle, minor) and of the individual figures remains incomprehensible. With its help these contro-

versial questions can receive satisfactory – and, moreover, perfectly simple – solutions. Further, we can then understand the scandalous fact that Aristotle 'failed to recognise' the fourth figure as such in spite of his allowing all its individual moods to be valid syllogisms. The corrections to the customary interpretations which will be proposed in Chapters III and IV, would, I am convinced, have occurred to any careful student of the *Prior Analytics* who had taken the trouble to put quite out of mind his knowledge of *traditional* syllogistic and to refrain from the unthinking replacement of Aristotle's own language by "the formulations of later logic which are both more familiar and easier to work with".[16]

The difference in question consists merely in this: in the systematic exposition of his syllogistic Aristotle never constructs propositions of the form "*S* is *P*" ("*A* ἐστὶν *B*"), but always writes "The *A* belongs to *B*"; more exactly, he always writes "The *A* belongs to all *B*" or "The *A* belongs to no *B*" or "The *A* belongs to some *B*" or "The *A* does not belong to some *B*". (τὸ *A* παντὶ τῷ *B* ὑπάρχει, τὸ *A* οὐδενὶ τῷ *B* ὑπάρχει, τὸ *A* τινὶ τῷ *B* ὑπάρχει, τὸ *A* τινὶ τῷ *B* οὐ ὑπάρχει: *A* 4, 26b3, 26a25, 36, 37.) Instead of "belong to" he also uses the relation "be said of"[17], and occasionally the expressions "*A* follows the *B*"[18] or "*B* is in *A* as in a whole" – the latter being defined as equivalent to "The *A* belongs to every *B*".[19] Thus in place of "All Greeks are men" Aristotle, in his syllogistic, would have written "Man belongs to all Greeks" or "Man is said of all Greeks" or "Man follows all Greeks" or "Greek is in man as in a whole". I say "*would have* written"; for in his systematic presentation of syllogistic Aristotle in fact formulates all syllogisms with variables and none with concrete terms.

All these expressions are as unnatural in Greek as they are in English. This assertion does not rest on our feeling for the Greek language; for Alexander of Aphrodisias (c. 200 A.D.) explicitly emphasized the fact in a passage of his commentary on the *Prior Analytics* which we shall shortly consider in more detail. Besides, Aristotle formulates his examples (except in the exposition of the theory, *APr. A* 4–7) *both* in ordinary language *and* in his technical terminology.[20] At the beginning of the book we find statements of the form "No pleasure is good" or "Some pleasure is good" (*A* 2, 25a6, 11) next to "Animal belongs to all man", "Man does not belong to all animal", "Man does not belong to some animal" (*A* 2, 25a25, 12). In the course of his further arguments in the *Analytics* Aris-

totle adheres to the technical idiom faithfully and without exception when dealing with propositions containing variables, while for propositions containing concrete terms he uses both the natural and the technical expressions. The syllogistic expansion of the enthymeme, for example, is written once in ordinary language:

> (All) ambitious men are generous; Pittakos is ambitious; Pittakos is generous (*APr. B* 27, 70a26–27);

and once in technical terminology:

> Paleness follows pregnant women, and paleness follows this woman (70a21 – the (invalid) conclusion is not so formulated: ὅτι κύει).

In the syllogism with concrete terms (*APst. B* 16, 98b5–10), which Łukasiewicz quotes (AS, p. 2), propositions with variables and propositions with concrete terms stand in immediate juxtaposition:

> Let *A* stand for 'deciduous', *B* for 'broad-leaved', *C* for 'vine'. (Then:) if to the *B* (properly "to all *B*") the *A* belongs – for everything broad-leaved is deciduous – and to the *C* (properly "to all *C*") the *B* belongs – for every vine (is) broad-leaved – to the *C* (properly "to all *C*") the *A* belongs and every vine is deciduous.[21]

From this example we can see quite clearly that the transition from the artificial idiom of logic, in which the propositions contain variables, to everyday – though still scientific – language, in which concrete terms occur, allows the 'natural' expression "*A* is *B*" to supplant surreptitiously the technical locution "*B* belongs to the *A*".

We cannot but ask *why* Aristotle introduced, and firmly adhered to throughout his systematic exposition, a mode of speech which *is* unnatural and which, as the cited passages show, he *felt* to be unnatural. None of the modern commentators, including Łukasiewicz, has even posed this question. If the difference is noticed, Aristotle's "usual practice" is invoked (Überweg, SdL[5], p. 331). Carl Prantl in his *Geschichte der Logik im Abendlande* mentions neither this nor the other differences between the Aristotelian and the traditional syllogism which we have noted, although he is much exercised to separate off Aristotle's logic from the "degenerate" traditional logic (Prantl I, pp. 348; 402). Alexander of

Aphrodisias, however, in the passage from his commentary mentioned above[22], not only pointed to the artificiality of Aristotle's language, but also tried to find an answer to the question which must now force itself on any conscientious interpreter: What could be the purpose of such artificiality? He presents three possible reasons; the first I do not understand, the second I hold to be completely correct, and the third to be demonstrably false. The first reason is supposed to be "because in this way the union of the terms (συναγωγὴ τῶν λόγων) is clear"[23]; the second "because in this way it is clearer which term is the subject and which the predicate"; the third "because in this way the first position in the proposition is held by the predicate which, being more general, is also first by nature". The first of these statements I do not, as I said, understand.[24]

The second points out that in the normal formulation, "*A* is *B*", both the linked terms are in the nominative case, so that to distinguish subject from predicate we depend upon the conventional order of the terms in the proposition – a convention which, in Greek as in English, is sometimes violated.[25] The formulae "The *A* is said of all *B*", "The *A* belongs to all *B*" and the other idioms preferred by Aristotle have this in common: in all of them the predicate is always in the nominative, the subject in the dative or (for the Greek κατηγορεῖσθαι and λέγεσθαι) in the genitive. While "*A* is *B*" may be compared to a balance, both pans of which hold a term, the picture, if any, suggested by all Aristotle's formulae is of some *ordered* process ("belong to", "follow"). I offer here the hesitant conjecture that the tendency which has continually reappeared in the history of logic, not least in more recent times, of conceiving a judgment as an equation, or even as an expression of identity[26], derives a good deal, if not all, of its force from the purely conventional wording of the schema "*S* is *P*"; this persuasive imagery is shunned by Aristotle's artificial terminology no less than by the symbolism of modern mathematical logic, where singular judgments appear as $\phi(a)$, and the formula $\phi(x)\rightarrow$ $\rightarrow\psi(x)$ expresses with similar clarity the order of the propositional connexion in a universal judgment. Thus Aristotle was induced to deviate from ordinary language because he wanted his linguistic expression to reveal with all possible clarity the *logical structure* of the propositions which enter the syllogism as premisses or conclusion.

Aristotle goes yet further in his attempt to illuminate the logical structure of propositions by their linguistic formulation: he introduces (*APr.*

A 41, 49b14–31) as an equivalent and clearer formulation of the proposition "The *A* belongs to every *B*" the sentence "The *A* belongs to all to which *B* belongs". This is yet further distant from the normal expression "All *B* is *A*". The new formula is close to the formal implication "For all x: if *B* belongs to x then *A* belongs to x", symbolically "$(x)\ (Bx \rightarrow Ax)$", which is of course the modern substitute for the traditional "All *B* is *A*". It is probable that Aristotle first developed this formulation *after* he had written his systematic exposition of syllogistic, in which propositions of the type "*A* belongs to all *B*" appear. It is only in his modal logic (*APr.* *A* 8–22) that he adopts this idiom (e.g. *A* 13, 32b25–29), and this is clearly later than the syllogistic theory of *A* 1–6 which it presupposes. For the present argument, however, it is enough if this example again reveals how deliberately Aristotle chose his language.[27]

Finally, Alexander's last statement can easily be refuted. He conjectures as a possible reason for Aristotle's technical mode of speech, the desire to grant the (more general) predicate term the first place in the proposition to which its 'natural priority' entitles it. But even in his new terminology Aristotle inverts the order of the terms on occasion, and for "The *A* belongs to the *B*" writes "To the *B* belongs the *A*"; indeed this is quite frequent: e.g. *APr.* *A* 2, 25a15-17; *A* 5, 27b3, 15, 18; *A* 6, 28a20, 25, b25; *A* 7, 29a39, b10, 14; *B* 12, 62a31; *B* 13, 62b14, 16, 18; *APst.* *A* 13, 78b2; *A* 26, 87a4: we have already met it in the example of the broad-leaved plants (*APst.* *B* 16, 98b6).

§ 5. The Aristotelian Form of the Syllogism

At the beginning of this chapter we set out the customary example of an 'Aristotelian' inference:

All men are mortal
Socrates is a man
―――――――――
Socrates is mortal.

We have already moved a considerable distance away from this. We saw that in his systematic presentation of the theory of the syllogism Aristotle only produces syllogisms which have the form of an "If ... then ..." proposition. Secondly, he uses (indeed was the first to use) variables (*A*, *B*, *C*...) in the place of concrete terms. Thirdly, special rules govern

the range of possible arguments for these variables. And finally to express the relations which hold between the variables, Aristotle avoids the usual copula of traditional logic and fashions different formulae. An Aristotelian syllogism, then, is a proposition of the form:

> If the *A* belongs to all *B* and the *B* belongs to all *C*, then the *A* belongs to all *C*.

This would be the Aristotelian counterpart of the traditional *Barbara*. However, Aristotle's actual formulation of this syllogism (*APr. A* 4, 25b37–39) is still slightly different. The first difference is insignificant: instead of the expression "belong to", which elsewhere preponderates, Aristotle employs the equivalent "be said of". We need consider this no further. The second difference is important: one word is added to the conclusion; it reads:

> "then the *A necessarily* belongs to all *C*".

The addition consists of the word "necessarily" (ἀνάγκη). The task of the following chapter is to investigate the difficult question of what this word means in Aristotle's logic.

NOTES

1.
$$\begin{array}{llllllll} \text{I.} & \begin{array}{c} MP \\ SM \\ \hline SP \end{array} & \text{II.} & \begin{array}{c} PM \\ SM \\ \hline SP \end{array} & \text{III.} & \begin{array}{c} MP \\ MS \\ \hline SP \end{array} & \text{IV.} & \begin{array}{c} PM \\ MS \\ \hline SP \end{array} \end{array}$$

Controversy surrounds the question why Aristotle does not distinguish the *fourth* from the *first* figure and only mentions the valid moods of the fourth figure, treating them as additional moods of the first, in two (later) supplementary chapters. The question will be discussed in Ch. IV (esp. § 25).

2. These names, or at least their close relatives, are first found in the *Summulae Logicales* of Petrus Hispanus (Pope John XXI, ob. 1277). Cf. I.M. Bocheński, *Formale Logik* (hereafter cited as: FL), pp. 244 sqq., English translation (hereafter: HFL), pp. 210 sqq.
3. In the language of mathematical logic. Read: "If *MeP* and *SiM*, then *SoP*".
4. *Barbara*, *Celarent* primae, *Darii Ferio*que;
Cesare, *Camestres*, *Festino*, *Baroco* secundae;
tertia grande sonans recitat *Darapti*, *Felapton*,
Disamis, *Datisi*, *Bocardo*, *Ferison*; quartae
sunt *Bamalip*, *Calemes*, *Dimatis*, *Fesapo*, *Fresison*.
5. E.g. H. Maier, *Die Syllogistik des Aristoteles* (1896–1900; hereafter cited as: SdA)

II, 1, p. 74; F. Überweg, *System der Logik*, (5th ed. 1882; hereafter: SdL[5]), p. 331. Characteristically, C. Prantl passes over this distinction in silence in his *Geschichte der Logik im Abendlande* (I 1855, II 1861; hereafter: Prantl).

6. J. Łukasiewicz, *Aristotle's Syllogistic from the standpoint of modern formal logic* (1951; 2nd ed. 1957; hereafter: AS), pp. 1 sqq. (The page-numbers of all my citations from this book apply to the first and second editions alike.)
7. A.N. Whitehead and B. Russell, *Principia Mathematica* I (1910; hereafter: PM), p. 28: "Syllogisms are traditionally expressed with 'therefore' as if they asserted both premisses and conclusion. This is, of course, merely a slipshod way of speaking, since what is really asserted is only the connection of premisses with conclusion."
8. On this cf. my paper 'Aristotle and syllogisms from false premisses', *Mind* **68** (1959), 186–192 (See Appendix, below p. 196).
9. So Łukasiewicz, AS, p. 21.
10. W.D. Ross, *Aristotle's Prior and Posterior Analytics* (1949; hereafter: APPA), p. 289.
11. However, Aristotle talks here not of 'things' (πράγματα) but of 'beings' (ὄντα) it is at least possible that he is giving in this passage a tripartite classification of beings: one class (the individuals) is defined by the fact that its members (or rather their names) can only be quartered in the *subject* place of a meaningful proposition; the second (the categories) by the fact that its members (their names) can occur only in the *predicate* place; members of the third class – terms proper – may appear in *both* places of a true proposition.
12. Bocheński (FL, p. 81; HFL, p. 70) supposes the reason why Aristotle restricted terms to members of the 'middle class' to be the logical fact that "the technique of the syllogism requires that each outer term occur at least once as predicate". But this is not a logical fact: in figures I and II *S* occurs only as subject.
13. The beginning of *A* 27 makes this quite clear; cf. esp. 43a20–24: πῶς δ' εὐπορήσομεν πρὸς τὸ τιθέμενον ἀεὶ συλλογισμῶν, καὶ διὰ ποίας ὁδοῦ ληψόμεθα τὰς περὶ ἕκαστον ἀρχάς, νῦν ἤδη λεκτέον. οὐ γὰρ μόνον ἴσως δεῖ τὴν γένεσιν θεωρεῖν τῶν συλλογισμῶν, ἀλλὰ καὶ τὴν δύναμιν ἔχειν τοῦ ποιεῖν.
14. Aristotle says (*A* 28, 44a7 sq.) that this is an argument in *Celarent* (I), and that, with the aid of an element common to the set of predicates of *P* and the set of contraries of *S*, we could also construct a syllogism in *Camestres* (II) (*PaM* & *SeM* → *SeP*). Thus he tacitly – and legitimately – converts *PeM* to *MeP*; he makes the same conversion to transform *Fesapo* (IV) (*PeM* & *MaS* → *SoP*) into *Felapton* (III). Clearly the notion of a contrary *predicate*-set would suffice: there is no need to introduce the further notion of a contrary subject-set (οἷς αὐτὸ μὴ ἐνδέχεται παρεῖναι). Here the strikingly modern tendency to make do with a minimum of concepts and logical laws is apparent: it can be traced throughout the *Analytics* and appears at its clearest in the reduction of the second and third figures to the first.
15. My thanks are due to Paul Lorenzen for valuable comments on the questions treated in this section.
16. Maier, SdA II, 1, p. 74, n. 2.
17. E.g. *A* 2, 24b28; *A* 41, 49b22–25: "λέγεσθαι κατὰ τινός"; *A* 4, 25b37: "κατηγορεῖσθαι κατὰ τινός".
18. E.g. *A* 4, 26a2; 26b6; *A* 27, 43b4: "ἀκολουθεῖν"; *A* 27, 43b3, 17, 22, 30; *A* 28, 44a13; *B* 3, 56a20 etc.: "ἕπεσθαι". Further references in Bonitz' *Index*; and cf.

H. Steinthal, *Die Sprachwissenschaft bei den Griechen und Römern* I (2nd. ed., 1891), p. 222.

19. *A* 1, 24b26–30.
20. In his discussion of negation, contradiction and contrariety at *de Int.* 7 sq., 17b16 sqq., Aristotle always uses the copula; e.g. ἔστι Σωκράτης λευκός (17b28).
21. ἔστω γὰρ τὸ φυλλορροεῖν ἐφ' οὗ *A*, τὸ δὲ πλατύφυλλον ἐφ' οὗ *B*, ἄμπελος δὲ ἐφ' οὗ *Γ*. εἰ δὴ τῷ *B* ὑπάρχει τὸ *A* (πᾶν γὰρ πλατύφυλλον φυλλορροεῖ), τῷ δὲ *Γ* ὑπάρχει τὸ *B* (πᾶσα γὰρ ἄμπελος πλατύφυλλος), τῷ *Γ* ὑπάρχει τὸ *A* καὶ πᾶσα ἄμπελος φυλλορροεῖ (*APst. B* 16, 98b5–10).
22. Alexander, *In Aristotelis Analyticorum Priorum librum I commentarium*, ed. M. Wallies, Berlin, 1883, p. 54, 21–29: χρῆται δὲ τῷ κατὰ παντὸς καὶ τῷ κατὰ μηδενὸς ἐν τῇ διδασκαλίᾳ, ὅτι διὰ τούτων γνώριμος ἡ συναγωγὴ τῶν λόγων, καὶ ὅτι οὕτως λεγομένων γνωριμώτερος ὅ τε κατηγορούμενος καὶ ὁ ὑποκείμενος, καὶ ὅτι πρῶτον τῇ φύσει τὸ κατὰ παντὸς τοῦ ἐν ὅλῳ αὐτῷ ὡς προείρηται (cf. 53, 24). ἡ μέντοι χρῆσις ἡ συλλογιστικὴ ἐν τῇ συνηθείᾳ ἀνάπαλιν ἔχει. οὐ γὰρ ἡ ἀρετὴ λέγεται κατὰ πάσης δικαιοσύνης, ἀλλ' ἀνάπαλιν πᾶσα δικαιοσύνη ἀρετή. διὸ καὶ δεῖ κατ' ἀμφοτέρας τὰς ἐκφορὰς γυμνάζειν ἑαυτούς, ἵνα τῇ τε χρήσει παρακολουθεῖν δυνώμεθα καὶ τῇ διδασκαλίᾳ.
23. The word "συναγωγή" occurs in Aristotle in a logical sense only in the *Rhetoric* (*Γ* 9, 1410a22; *B* 23, 1400b26), where it means much the same as the logical proof of a proposition. In Alexander (cf. 109, 32: τοῦ μὲν *A* πρὸς τὸ *B* οὐδεμία συναγωγὴ γίνεται) it seems rather to mean the operation of uniting, or 'leading together', the two *outer terms* of given premisses into a conclusion. If this is so, Alexander is wrong: the συναγωγή can be *just* as clear when the copula is used, provided that the order of the premisses is reversed. On this cf. § 16.
24. Prof. W.H. Friedrich has pointed out to me the possibility of making the last two sentences *subordinate* to the first. The meaning would then be something like this: "Aristotle uses this artificial mode of speaking because it increases the clarity of the συναγωγή, since (1) subject and predicate terms appear more clearly, and (2) the predicate term moves to its rightful first place" – i.e. the advantage of Aristotle's idiom consists, to be more precise, of *two* advantages. It is only the fact that this reading of the passage, which grammar certainly permits, agrees so well with my own interpretation of Aristotle, which restrains me from embracing it unreservedly.
25. So occasionally in Aristotle, e.g.: Ζῷον μὲν γὰρ ὁ ἄνθρωπος ἐξ ἀνάγκης ἐστί (*APr. A* 9, 30a30).
26. E.g. Hamilton, Lotze, Bradley.
27. For the interpretation of *APr. A* 41, cf. Bocheński, FL, pp. 49, 92, 96; HFL, pp. 42, 80, 83.

CHAPTER II

LOGICAL NECESSITY

§ 6. ἀνάγκη, ἐξ ἀνάγκης, ἐνδέχεσθαι and Related Terms

It is readily seen that by adding the word "ἀνάγκη" to the conclusion of a valid syllogism Aristotle does not mean to make it into what traditional logic called an apodeictic proposition; for, according to the explicit doctrine of *APr. A* 12, 32a6–14, a proposition asserting that a predicate belongs necessarily to a subject can only be validly inferred when at least one of the two premisses is itself apodeictic.[1] This was quite clearly explained by Alexander (l.c. 20.30–21.9); he notes that some people actually had taken this "ἀνάγκη" as marking the necessity of the proposition which occurs as conclusion. Heinrich Maier too discussed the question at length and established that Aristotle makes a clear distinction between the necessity which always belongs to the conclusion of a valid inference as such, and the necessity which can sometimes occur as a modal operator on the final proposition of a syllogism; and that he never confounds these two operators (SdA II, 2, pp. 242–244).

Aristotle *defines* this difference (*APr. A* 10, 30b31–33 and 38–40; cf. *APst. B* 11, 94a21–27 and *B* 5, 91b14–17) as one between relative and absolute necessity (τίνων ὄντων ἀναγκαῖον–ἁπλῶς ἀναγκαῖον), more exactly between a "necessity which exists if or as long as something (else) exists" and a "necessity simpliciter". A proposition such as "All horses are animals" is in his view necessary in its own right, and may be formulated "Animal necessarily belongs to all horses"; on the other hand, a proposition such as "No white (thing) is a man", to use Aristotle's own example (*APr. A* 10, 30b33–40), is only necessary if, say, it is the case that all men are animals and no white (thing) is an animal.

It is of course not very satisfactory to define the difference between two kinds of 'necessity' in this way: first, because obviously the so-called 'simple' or 'absolute' necessity is itself necessary only in the context of some system of classification, and hence only *if* certain definitions are presupposed[2]; and secondly, because the nature of 'necessity τίνων

ὄντων', of 'relative' necessity, is not really clear. However, Aristotle's position can be worked out and made clear from his own words: a proposition which is apodeictic in its own right asserts in itself that its predicate necessarily belongs to its subject; the addition of ἀνάγκη to a conclusion is meant to show that the assertion it makes is necessarily *true if* the premisses are true. It is plain that these are two quite independent 'necessities' and that it can, for example, be necessarily true (in virtue of certain premisses) that a predicate belongs possibly to a subject, and also possibly true (in virtue of certain other premisses) that the predicate belongs necessarily to the subject. The latter would be the case if it could not be *inferred* from the premisses that the predicate does possibly not belong to the subject. We might therefore offer this as a brief account of Aristotle's position: the 'relative' necessity appearing in the formulation of a syllogism is conveyed not by the simple word "necessarily" ("ἀνάγκη"), but by the whole expression "If ... then necessarily"; whereas to signify 'absolute' necessity (ἁπλῶς ἀναγκαῖον) the word "necessarily" is used without qualification. The converse of course holds too: where the word "necessarily" is used in connexion with an "if", we have a case of relative necessity.

Particular difficulty threatens the distinction between 'relative' and 'absolute' necessity in modal logic, in cases where a proposition necessary in its own right appears as a conclusion and must therefore, like the conclusion of *Barbara* (I) in our example, also take the sign for 'relative' necessity.

Aristotle neatly evades these difficulties by choosing two different *expressions* for 'relative' and 'absolute' necessity: ἀνάγκη ὑπάρχειν for the former and ἐξ ἀνάγκης ὑπάρχειν for the latter (e.g. *APr. A* 10, 30a39: ἀνάγκη δὴ τὸ *A* τινὶ τῷ *Γ* ὑπάρχειν ἐξ ἀνάγκης). In propositions which contain only one of these operators, the distinction is not of course indispensable – and hence it is not always firmly adhered to: occasionally the simple "ἀνάγκη" stands for 'absolute' necessity (e.g. *A* 10, 30b28; *A* 11, 31b7). However, in chapters *A* 8–22 of the *Prior Analytics*, where Aristotle presents his modal syllogistic, I have counted 53 cases in which he formulates syllogisms containing a necessary premiss, and in only *two* of these have I found that he expresses the '*absolute*' necessity of premisses or conclusion by ἀνάγκη and not by ἐξ ἀνάγκης[3].

In the first edition of this book I made the further contention that in

A 8–22 "ἐξ ἀνάγκης" is reserved *exclusively* for 'absolute' necessity. This thesis has been attacked by Ackrill (*Mind* 71 (1962), 109 n.), who points to *A* 15, 34a7, 17 and 21. I concede at once that in 34a7 and 17 (where the modal operator governs propositions which are not conclusions of modal syllogisms) the context seems to suggest that "ἐξ ἀνάγκης" be taken in the sense of 'relative' necessity. In the more important passage, a21, where "ἐξ ἀνάγκης" appears in a conclusion, two interpretations are possible: that of Ross (APPA, p. 333) and Ackrill, which construes the necessity as 'relative', fits better the *preceding* text; the other, which takes "ἐξ ἀνάγκης" to express the 'absolute' necessity of the *two* premisses *and* the conclusion, would suit the *following* text better. Here too, however, the reading of Ross and Ackrill is prima facie the more natural.

It is worth noting that this very passage (*A* 15, 34a1–34) has recently been more than once discussed for its importance to Aristotle's modal logic. Von Wright, in his *Logical Studies* (1957, pp. 125–126) has treated it in detail in his attack on Łukasiewicz' interpretation (AS, pp. 138–139); and most recently Hintikka has shown, with explicit reference to the thesis of mine which is now under discussion, that a plausible interpretation can be developed which takes "ἐξ ἀνάγκης" both times in the sense of '*absolute*' necessity. Whether or not Ackrill's objection is conclusive must therefore remain an open question.

Aristotle has another way of preventing possible confusion of the operators: he chooses expressions other than the word "ἀνάγκη" to mark the 'relative' modality of the conclusion. Even in the assertoric syllogisms of *A* 4–7, where he generally symbolises 'relative' necessity (the only modal operator occurring there) by the addition of ἀνάγκη to the conclusion, he occasionally uses a different expression – for example the future ὑπάρξει, "will belong", appears at *A* 4, 26a2; *A* 5, 27a7, 10, 12; b19; *A* 6, 28b34; *A* 7, 29b10, 13. In Greek the future tense can express necessity, in particular the necessity of a consequence; Aristotle exploits this nuance especially often with the future of εἶναι.[4] Yet another way of conveying the 'relative' operator is "ἔσται συλλογισμός ..." (*A* 5, 27b3) or "γίνεται συλλογισμός ..." (28a2). Sometimes the phrase "ὑπάρξει ἐξ ἀνάγκης" occurs to indicate '*relative*' necessity (*A* 6, 28a19) and even the compound pleonasm "ἔσται συλλογισμὸς ὅτι ... ὑπάρξει ἐξ ἀνάγκης" (28a27).

It is a convincing indication of the care with which Aristotle chose his

words that, by contrast to this, in the whole of the *modal* logic he does not once use "ἐξ ἀνάγκης" for 'relative' necessity: precisely because this expression there signifies 'absolute' necessity. Whereas in the assertoric chapters, *A* 4–7, 'relative' necessity is denoted, in the great majority of cases, by the phrase "εἰ ... ἀνάγκη ...", in the modal logic Aristotle uses his equivalent expressions almost without exception in cases where the conclusion itself is already governed by an 'absolute' operator. In such propositions the future of "ὑπάρχει" denotes 'relative' necessity, the addition of "ἐξ ἀνάγκης" always signifies 'absolute' necessity. This becomes clear beyond all question in cases where one proposition which is, in Aristotle's sense, 'absolutely' necessary and one which is not together entail a conclusion which is not an 'absolutely' necessary proposition: here Aristotle explicitly says: "ὑπάρξει, ἀλλ'οὐκ ἐξ ἀνάγκης" (*A* 10, 31a11–14; *A* 11, 31b1) – the future of "ὑπάρχει" affirms 'relative' necessity, the addition of "ἀλλ'οὐκ ἐξ ἀνάγκης" denies 'absolute' necessity. Examples of this use of the future of "ὑπάρχει" in the modal logic may be found at: 30b27, 36; 31a10, 33, 36, 40; 37b26; 38a19 (I give only the page and line number of Bekker's edition). The expression "ἔσται συλλογισμός ..." or "γίνεται συλλογισμός ..." replaces "ἀνάγκη" at, for example: 28a27; 32b39; 33b39; 35b40; 36a4; 37b27, 33; 38a20. At times we find "φανερὸν ὅτι ..." in the case of 'perfect' inferences (cf. § 12): e.g. 30a22; 31a13; 33b35. The expression "ἀνάγκη" for the 'relative' operator of the conclusion of a valid syllogism, which predominates in the assertoric logic and is indeed the obvious phrase to use, occurs only six times in the whole of the modal section (30a39; 33a26; 34a35; 34b21 – τούτων τεθέντων ἀνάγκη – 36a10; 35). In four places the 'relative' operator is wholly unexpressed: 33a24; 34a39; 38a34; 39a34–35.[5]

These facts about Aristotle's linguistic practice show plainly that not only was he quite clear about the distinction between what he calls 'relative' and 'absolute' necessity, but he also took terminological care, by the choice of different expressions, to avoid the confusion between the two which is particularly liable to occur in modal contexts. He did not succeed in every case. In two places, as we saw, he symbolises the 'absolute' necessity of a conclusion by the expression elsewhere reserved exclusively for 'relative' necessity (31b17 and 36); and in a further passage he formulates the proposition "It is possible that the *A* necessarily belongs to all *B*, and (choosing other terms for *A* and *B*) that it necessarily

belongs to no *B*" – which on his principles should run "ἐνδέχεται τὸ *A* τῷ *B* παντὶ καὶ μηδενὶ ὑπάρχειν ἐξ ἀνάγκης" – in a highly misleading, or rather downright false, manner: "καὶ γὰρ παντὶ ἀνάγκη τὸ *A* τῷ *B* καὶ μηδενὶ ὑπάρχειν" (39b3-4), a sentence which can only be translated "For it is necessary that the *A* belongs to every *B* and that it belongs to no *B*". Here Aristotle makes use of the notion of 'relative' *possibility* – a term which he never explicitly defines, as he does 'relative' *necessity*, but which he uses constantly, even in the assertoric chapters, wherever he is proving that a given pair of premisses cannot yield a syllogism "because nothing necessary follows from the fact that what the propositions assert to hold does hold".[6] To the assertion that nothing 'relatively' necessary follows, he immediately adds, as if it were equivalent in meaning, a second proposition[7] that, relative to the propositions being tested for their concludence (in this case, the propositions "*A* belongs to all *B*" and "*B* belongs to no *C*") it is possible ('relatively' possible) that *A* belongs to all *C* and that *A* belongs to no *C*. To *negate* the 'relative' necessity of a conclusion – or rather, since the conclusion is by definition a proposition necessary relative to certain premisses, to negate the necessity of a proposition relative to other given propositions, we must assert the possibility, relative to the same premisses, of the *negation* of this proposition. In this passage Aristotle quite correctly asserts 'relative' possibility by means of the expression "ἐνδέχεται ὑπάρχειν". He commits a mistake when in the corresponding part of our modal example (39b3–4) he *omits* this quite indispensable expression.

This case apart, Aristotle chooses his words for "possibility" too with care and deliberation. This is shown by the way in which he deploys the various expressions which ordinary language has for "possibility". The need to use the different offerings of everyday speech to mark off different kinds of necessity and possibility naturally first struck Aristotle when he came to discuss modal logic. In *A* 3, just as the two expressions "ἀνάγκη" and "ἐξ ἀνάγκης", later distinguished, are used as equivalents (εἰ δὲ **ἐξ ἀνάγκης** τὸ *A* παντὶ ἢ τινὶ τῷ *B* ὑπάρχει, καὶ τὸ *B* τινὶ τῷ *A* **ἀνάγκη** ὑπάρχειν: 25a32–33), so too Aristotle does not here discriminate between "ἐνδεχέσθαι" and "ἐγχωρεῖν" (εἰ γὰρ **ἐνδέχεται** μηδενὶ ἀνθρώπῳ ἵππον, καὶ ἄνθρωπον **ἐγχωρεῖ** μηδενὶ ἵππῳ: 25b9). Even in the modal syllogistic proper "ἐγχωρεῖ" is at times used as a complete synonym for "ἐνδέχεται": e.g. *A* 10, 30b15: εἰ γὰρ τὸ *A* μηδενὶ τῷ *Γ* **ἐνδέχεται**, οὐδὲ τὸ *Γ* οὐδενὶ

τῷ *A* ἐγχωρεῖ.[8] However, where both modal operators, 'relative' and 'absolute' possibility, occur together in a proposition, Aristotle *had* to try to vary his language, in order to avoid confusion – particularly since he does not distinguish 'absolute' and 'relative' possibility as explicitly as he did 'absolute' and 'relative' necessity. When this case first occurs, Aristotle uses the familiar "ἐνδέχεται" for 'relative' possibility, which he places at the beginning of the proposition, and employs "ἐγχωρεῖ" for 'absolute' possibility: *A* 9, 30a27–28: ἐνδέχεται γὰρ τοιοῦτον εἶναι τὸ *B* ᾧ ἐγχωρεῖ τὸ *A* μηδενὶ ὑπάρχειν. Later he rightly judged it better to reserve "ἐνδέχεται" for 'absolute' possibility, and to choose a different expression for the 'relative' operator. Thus, 'relative' possibility is denoted by "ἐγχωρεῖ" at 37b7; 38a38; by "συμβαίνει" at 38a29[9]; and by "οὐδὲν κωλύει" at 30b30; 34b11 and 38a39.

Aristotle had tried to circumvent confusion between 'relative' and 'absolute' *necessity*, an everpresent danger in his modal syllogistic, by sometimes expressing 'relative' necessity (the less frequent operator) by means of paraphrases in which the word "ἀνάγκη" and its derivatives do not appear at all, and sometimes giving no symbol for it whatever, but leaving to the reader the simple task of supplying one for himself. He hoped to avoid confusion between 'relative' and 'absolute' *possibility* – which he never explicitly differentiates – in just the same way: for 'relative' possibility, the less frequent operator, he chooses expressions different from the "ἐνδέχεται" which was introduced for 'absolute' possibility; and sometimes, as in our example, (39b3) he simply omits any sign for the 'relative' modality.[10] This practice of simply leaving out the 'relative' operator is logically admissible in the case of 'relative' necessity; but it is not in the case of 'relative' possibility. The following section will show why this is so; the present considerations aim only to make it plain that Aristotle's efforts to distinguish 'absolute' from 'relative' modal operators embraced not only necessity, but possibility as well; and that clear traces of these efforts are in his text.

§ 7. Logical Objections to Aristotle's Distinction between Two Types of Necessity

These peculiarities of Aristotle's language show beyond any doubt that he distinguishes clearly between a 'relative' and an 'absolute' necessity;

and we have explained how he defines the difference between the two kinds of necessity. I shall now proceed to raise some objections against the way in which Aristotle talks about necessity. My claim is that Aristotle's distinction between 'relative' and 'absolute' necessity depends on a profound misunderstanding of the phenomenon he calls 'relative' necessity.

Whenever Aristotle distinguishes between ἀναγκαῖον τίνων ὄντων and ἀναγκαῖον ἁπλῶς ('relative' and 'absolute' necessity), he always takes the distinction as one between the different species of necessity, which may in certain cases belong to one and the same proposition – the proposition concluding certain valid inferences. The conclusion of *Darii* (I) "ἀνάγκη τὸ *A* τινὶ τῷ *Γ* ὑπάρχειν" (*A* 4, 26a25), and the conclusion of a modal *Darii* with one apodeictic and one assertoric premiss "ἀνάγκη δὴ τὸ *A* τινὶ τῷ *Γ* ὑπάρχειν ἐξ ἀνάγκης" (*A* 9, 30a39), are distinguished (in Aristotle's language) merely by the fact that the first proposition contains *only* the 'relative' operator, while the second contains both 'relative' and 'absolute' operators. A proposition of the form "*A* belongs to all *B*" has the 'relative' operator, in Aristotle's view, if it is the conclusion of a valid syllogism; it has the 'absolute' operator if it is an apodeictic proposition in its own right. Such apodeictic propositions may occur as premisses of the syllogisms which Aristotle investigates in his modal logic: they may also enter these syllogisms as conclusions, and they take in addition the 'relative' operator. Aristotle's way of talking about necessity obviously hinges on the fact that 'absolute' and 'relative' necessity are different species of necessity, and that one and the same proposition, say "*A* belongs to all *C*", may contain them both together, or one of them or none at all. It is because of this assumption that Aristotle's distinction, plain and appropriate though it may at first appear, nevertheless remains unsatisfactory, and indeed can be proved to be logically incorrect.

No doubt it makes sense – and even in a sense is true – to say that the conclusion of a valid argument is 'relatively' necessary; and no doubt it makes sense – and in a sense is true – to say that the conclusion of a valid argument in certain well-defined cases which Aristotle investigates in his modal logic, is an 'absolutely' necessary proposition. However, that is not to say that it is one and the same subject (namely, the proposition in question) of which 'absolute' and 'relative' necessity are in such cases predicated. For the 'relative' operator governs the conclusion *qua* con-

clusion, while the 'absolute' operator governs the conclusion, not *qua* conclusion, but taken as an independent proposition. Since we can quite legitimately say of a proposition that it is the conclusion of a valid inference, it is certainly very tempting to suppose that the 'relative' operator too belongs to the proposition *qua* proposition (for it is true that it is a conclusion, and true that all conclusions of valid arguments are 'relatively' necessary propositions). However, this natural presentation of the facts has an unfortunate characteristic: it is *systematically misleading*. The phrase "systematically misleading" is not meant as a vague accusation; it is meant to describe Aristotle's terminology: in philosophy it is not infrequently found that some fact, patently true but difficult to describe with precision, has been expressed in such a way that what was intended as a simple description entails consequences which the fact itself by no means warrants. Similar cases of systematically misleading formulations will come to our attention later (cf. p. 94 sq., p. 188, n. 28). What are the unwarrantable consequences in our case?

Aristotle maintains[11] that in 'absolutely' necessary propositions the predicate belongs necessarily to the subject. His standard example is the proposition that the angle-sum of a triangle is necessarily equal to two right angles. Here the predicate "has an angle-sum of two right angles" belongs necessarily to the subject "triangle"; that is, it is impossible for anything both to be a triangle and to have an angle-sum other than two right angles. It is the same, according to Aristotle, in the case of the proposition that a man is necessarily an animal. Now Aristotle distinguishes '*relatively*' from '*absolutely*' necessary propositions by stipulating that the former are only necessary if certain conditions are fulfilled. But this mode of expression implies that a 'relatively' necessary proposition, when the conditions relative to which it is necessary are fulfilled, must be necessary in the same sense as an 'absolute' proposition is necessary, as it were, from the cradle. And this is obviously not the case. For – to use an Aristotelian example[12] – if it is true that all horses are asleep and that every horse is necessarily an animal, then, by *Darapti* (III), it must be the case that some animals (as a matter of fact) are asleep; but this necessity is fundamentally different from the necessity with which all triangles have an angle-sum of two right angles: even if some animals are as a matter of fact asleep, they could still wake up at any time; a triangle, on the other hand, can never change its angle-sum. Rather, in Aristotle's

'absolutely' necessary propositions the predicate belongs *necessarily* to the subject, whereas in 'relatively' necessary propositions, even if the conditions are satisfied, it only ever follows that the predicate belongs *as a matter of fact* to the subject. It is necessarily true that, if I meet in Göttingen a friend whom I thought was abroad, then my friend is in Göttingen; but that does not make his stay in Göttingen necessary – he may have broken his journey there by mere caprice.

If it were true, as Aristotle's distinction between 'relative' and 'absolute' necessity assumes, that a 'relatively' necessary proposition, when the conditions for its necessity are satisfied, possesses the same necessity as that which an 'absolutely' necessary proposition has unconditionally, then every true proposition would be an 'absolutely' necessary proposition in Aristotle's sense of the word. For every proposition can, trivially, be treated as its own condition; and, on the basis of the logical law "If *A*, then *A*", we can write "If Charles is well, then Charles is well". The consequent satisfies Aristotle's requirements for 'relatively' necessary propositions: it must be true if the antecedent (in this case the proposition itself) is true. But we may not follow Aristotle and transfer the necessity from the *truth* of an *implication* to the *fact* which its *consequent* expresses: if that were legitimate, then nobody who was well would need to worry about preserving his health.

Aristotle's distinction between 'relative' and 'absolute' necessity is systematically misleading: it ignores the difference between the necessary truth of an implication and the necessity of the fact expressed by its consequent.

A corresponding error plainly underlies Aristotle's notorious assertion that statements about the future are not subject to the law of excluded middle. This doctrine, presented in the ninth chapter of the *De Interpretatione*[13], has caused just astonishment in view of the uncompromising spirit with which in the *Metaphysics* (Γ 7, 8) Aristotle defends the uninhibited validity of the law against the attacks of Heraclitus and Protagoras.

Aristotle's argument is this: by the law of excluded middle it is necessary that one at least of the propositions "A sea-battle will occur tomorrow" and "No sea-battle will occur tomorrow" is true. This is equated with the assertion that already today either the sea-battle is necessary or its non-occurrence is necessary. But then every Council of War would be pointless: for it is absurd to deliberate about something which

is determined by ineluctable necessity. Aristotle therefore prefers to reject the application of the law of excluded middle to future statements, rather than to subscribe to a fatalism of this sort. – But from the fact that it is necessary that tomorrow the sea-battle will either occur or fail to occur, it does not of course follow that the sea-battle itself (or its non-occurrence) is necessary. Aristotle mistakenly thinks that the two propositions mean the same, because he does not distinguish between the necessary truth of an implication and the necessity of the fact stated by its consequent: it is necessarily true that, if a sea-battle is going to be fought tomorrow, then a sea-battle will occur tomorrow – but that does not turn the sea-battle into a necessary event.

But there is worse to follow: the systematically misleading terminology which calls the conclusions of valid syllogisms 'relatively' necessary, leads to a contradiction within Aristotle's text itself: Aristotle says (*APr. A* 10, 30b31–40) that from the premisses "*B* belongs necessarily to all *A*" and "*B* belongs to no *C*" it necessarily follows (by a modal *Camestres* (II) with a necessary first premiss) that "*A* belongs to no *C*", but not that "*A* belongs necessarily to no *C*". Aristotle illustrates this by taking a syllogism with concrete terms in place of variables: "If animal belongs necessarily to all men and animal belongs to no white, then man belongs to no white". He concludes: "The conclusion will be necessary if the premisses are true (τούτων μὲν ὄντων) but not absolutely necessary." In the case of the concrete example "Then man will belong to no white, but not necessarily. For it is possible for a man to become white, but not as long as animal belongs to no white"[14]. The contradiction lies in the last sentence: if we substitute for "as long as" the word "if" (equivalent in this context), and for "It is not possible for a man to become white" the synonymous "Man belongs necessarily to no white", then we are back with the necessity of the proposition "Man belongs to no white", which Aristotle has just expressly rejected. It is not correct to say that man *can* belong to no white as long as animal belongs to no white. Of course man even then *can* belong to a white – otherwise it would be quite impossible for any animal to become white (and the second premiss would be, contra hypothesin, '*absolutely*' necessary). It is true only that man *belongs* to no white thing as long as the premisses hold.

However, the necessity which Aristotle discusses under the misleading name of "relative necessity", is not simply a chimaera. Nor is it merely

an alias for the 'absolute' necessity which he ascribes to certain propositions of the form "*A* belongs/does not belong to all/some *B*" – in which case both types of necessity would coincide where the conclusion of a valid syllogism is itself such an 'absolutely' necessary proposition. Aristotle was right to *emphasise* the distinction between the two occurrences of necessity here – but he gave a faulty *definition* of the distinction when he took it as one between two varieties of necessity both belonging to formally identical propositions.

Moreover, it can be shown that, despite his misleading *definition* of so-called 'relative' necessity, Aristotle was logician enough to *use*, within his syllogistic theory, a different, and perfectly correct, notion of the controversial operator. This difference comes out most clearly in the procedure we have already mentioned (p. 20) by means of which Aristotle proves that a given syllogistic formula is not a valid syllogism. We shall discuss the procedure in detail in § 31; here a preliminary account must suffice. Aristotle regards a syllogistic formula as *invalidated* if terms can be produced which, when substituted for *A*, *B*, and *C* in the premisses and conclusion, make the former true and the latter false. Since the disproof of a proposition and the proof of its negation are one and the same thing, the negation of the proposition used in the disproof must be equivalent to the original proposition.

An example will make this clearer: Aristotle disproves (*A* 5, 27a18–20) the validity of the 'syllogism' (syllogistic formula) "If *B* belongs to all *A* and to all *C*, then *A* belongs to all *C*", by asserting that "there are terms *A*, *B*, *C*, which make the premisses "*B* belongs to all *A*" and "*B* belongs to all *C*" true, and the conclusion "*A* belongs to all *C*" false." This proposition is itself proved by producing three such terms, namely "animal" for *A*, "substance" for *B* and "number" for *C* – it would be clearer for us to take, say, "animal", "horse", and "man". If the production of three such terms can in fact *disprove* a formula such as the present 'syllogism', then (still following Aristotle) a syllogism must itself *assert* that such terms do not exist; that is, it must assert that, for all terms *A*, *B*, *C*, if the substitution of these terms for *A*, *B*, and *C* in the premisses makes the premisses true, then their substitution in the conclusion must also make the conclusion true. The addition of the word "necessary" ("ἀνάγκη"), which we are doing our best to understand, then denotes not the necessity of the conclusion alone, but the necessary truth of the whole compound

proposition which forms an Aristotelian syllogism. And the necessary truth of the whole proposition means, still following Aristotle's practice in his disproofs of formulae, precisely its *universal validity* for all terms which satisfy the syllogistic axioms (1)–(3) of page 7. That is to say, the sign for necessity is, in modern terms, a *universal quantifier* ranging over syllogistic term-triples, (*A*, *B*, *C*).

Had Aristotle wanted to direct his *method* of disproof along the path laid down by his *definition* of 'relative' necessity (according to which the conclusion of a valid syllogism must be a 'relatively' necessary proposition, and hence, if the premisses are true, an 'absolutely' necessary proposition), then *Barbara* could be adequately disproved by the production of three terms the substitution of which for *A*, *B*, *C* would make the premisses true but the conclusion non-necessary; – and it is obvious that every valid assertoric syllogism could easily be 'disproved' by this method.

Syllogistic necessity, then, is not, as Aristotle maintains, the 'relative' necessity of, say, the *A*'s belonging to all *C*, but the ('absolute') necessity that the whole implication between premisses and conclusion is a true proposition. And "necessarily true" means (as Aristotle's method of disproof shows) nothing more than truth in all possible cases, that is, for any substitution of concrete terms for the variables *A*, *B*, *C*.

The equation of "necessary" and "always true", which can be abstracted from Aristotle's *practice*, but not from his *definition*, is by no means foreign to him. "τὸ ἀεί" (that which always is as it is) and "that which necessarily is" are for him – in one[15] of the meanings of "ἐξ ἀνάγκης ὄν" – one and the same.[16] We may compare Leibniz' dictum that necessary truths are those which are true in all possible worlds; or Kant's statement: "Necessity and strict universality are thus sure marks of *a priori* knowledge, and are inseparable from one another. Since, however, ... the unlimited universality of a judgment can sometimes be more convincingly proved than its necessity, it is advisable to use the two criteria separately – each being in itself infallible" (*Critique of Pure Reason*, *B* 4). And later: "The schema of necessity is the existence of an object at all times" (ib. *B* 184). The reference to the passages in Aristotle and to the two statements of Kant is not inappropriate in view of the objection often raised against enquiries of this sort, that they measure Aristotle and his logical theories by canons utterly strange to him – canons which

indeed can only be drawn from the remote province of modern mathematical logic.

We have seen that Aristotle's 'relative' necessity is not the necessity of the *conclusion* of a valid inference, but the necessary truth of the compound proposition which, at the beginning of the book, we established an Aristotelian *syllogism* to be. The modal operator we are investigating is thus not an operator on one of the three propositions of the syllogism (the conclusion); it governs the compound "If ... then" proposition considered as a whole. To clarify the nature of this operator we are able to draw on Aristotle's assertion that the primary sense of the word "necessity" is "universality", "universal validity" – "immutability". To say that a syllogism in *Barbara* (1) is, in Aristotle's sense, necessary, is to say that a proposition of this form is 'always true'; and this in turn means that it is true whatever concrete terms replace the variables *A*, *B*, *C*. Logically speaking this necessity operator, which Aristotle rightly, if for the wrong reasons, distinguishes from the operator on an apodeictic proposition of the sort which at times occurs as the conclusion of an inference, corresponds to the universal quantifier of formal logic: "(*For all terms A*, *B*, *C*): if the *A* belongs to all *B* and the *B* belongs to all *C*, then the *A* belongs to all *C*." After these explanations it is neither unreasonable nor confusing to replace the quantifier and write: "Necessarily true: if *A* belongs to all *B* and *B* belongs to all *C*, then *A* belongs to all *C*." To Aristotle's usual formulation, "εἰ τὸ *A* παντὶ τῷ *B* καὶ τὸ *B* παντὶ τῷ *Γ* ὑπάρχει, ἀνάγκη τὸ *A* παντὶ τῷ *Γ* ὑπάρχειν", we can only make the seemingly pedantic objection that he would have done better to put the "ἀνάγκη" at the beginning, and to have written: "ἀνάγκη, εἰ τὸ *A* παντὶ τῷ *B* καὶ τὸ *B* παντὶ τῷ *Γ* ὑπάρχει, τὸ *A* παντὶ τῷ *Γ* ὑπάρχειν". That this proposal only *seems* pedantic is sufficiently proved by the fact that a logician of Aristotle's calibre fell victim to the specious suggestion of his own formulation, and believed that the operator expressed by ἀνάγκη only governed the conclusion which it immediately precedes.

We have reached the following results: Aristotle tried to make palpable the distinction between the necessity with which the conclusion of a valid inference 'follows', and the necessity which can belong to a conclusion if it is an apodeictic proposition, by defining the two as different species of necessity, which can in certain cases belong to one and the same proposition (a conclusion of the type mentioned). We have shown that in these

two cases the operator governs propositions of fundamentally different forms; in the one case "A belongs to B", in the other "If A, then B".[17]

§ 8. Some Possible Criticisms

Let us assume that the operator which Aristotle called 'relative' necessity must be interpreted as a universal quantifier over the variables of a syllogistic formula. We could still suppose that Aristotle was nevertheless right to treat the operator as governing the conclusion alone and not the whole syllogism, since he wanted to say that, for any terms A and C, it is the case, for example, that A belongs to every C if it is the case that A belongs to all B and B to all C. We would then have a proposition of the form: If A belongs to all B and B belongs to all C, then for all possible values of A and C it is the case that A belongs to all C. However, there is an objection to this formulation: the conjunction of the premisses is not a true or false proposition, since the premisses are empty propositional schemata which only become propositions when concrete terms such as "man", "animal", "horse", replace the variables A, B, and C.

We might perhaps reply to this by pointing to the fact that in mathematical logic a propositional *schema* (propositional *function*, open sentence) can be asserted by the assertion of its universal closure[18], so that the proposition we wrote above is simply an alternative formulation of the proposition: If for all terms A, B, C it is the case that A belongs to all B and B belongs to all C, then for all terms A, C, it is the case that A belongs to all C. This proposition is in fact a law of logic; that is, it is true whatever terms are substituted for A, B, C. However it is not the syllogism *Barbara* (I) which Aristotle wants to formulate. It asserts the following logical truth: If all sets of three terms are related in such a way that the first-named term is superordinate to the second and the second-named is superordinate to the third, then for all pairs of terms the first-named is superordinate to the second. It is clear that this proposition is true if there are any three terms which do not stand in this relation. For in that case the antecedent is false, and an "If ... then ...." proposition is true if its antecedent is false. The proposition in question is 'weaker' than *Barbara*, in the sense that it is entailed by but does not entail *Barbara*. The logical relationship between these two propositions is plainly the same as that between "All men are mortal" and "If everything is a man then

everything is mortal". The last proposition is true if there is anything at all which is not a man; but in this case the former proposition need not be true: for there could still be things which are men without being mortal, in which case the proposition "All men are mortal" would clearly be false.[19]

Now, as we have already noted, Aristotle occasionally formulates syllogisms with concrete terms in the place of the variables *A*, *B*, *C*. Here too he says that the proposition which is the conclusion of such a concrete syllogism is 'relatively' necessary (e.g. *A* 10, 30b35–40). In general a conclusion, in Aristotle's view, is a proposition which "follows from certain premisses with (relative) necessity" (*APr. B* 2, 53b18–19: τὸ μὲν γὰρ συμβαῖνον ἐξ ἀνάγκης τὸ συμπέρασμά ἐστιν). Has Aristotle's attribution of 'relative' necessity to the conclusion, that is to the proposition which occurs as the conclusion, more justification here where concrete terms stand in place of variables? One might at first be inclined to think so. For in these cases the premisses are no more propositional schemata than the conclusion; they are genuine propositions which contain no variables not bound by universal quantifiers (to translate into the language of formal logic) and no variables at all if we consider the wording in Aristotle's text.

The previous objection would not apply here. Would it not be reasonable then to call the conclusion of such a syllogism a ('relatively') necessary proposition, since it must be a true proposition *if* the premisses are true? To answer this question let us first compare two propositions, the first an example which Aristotle himself offers, the second a thing of our own. The first proposition is found at *APr. A* 10, 30b32–40; we have already met it. It contains an 'absolute' operator which we can here disregard.

(1) If animal belongs to all men and animal belongs to no white, then man belongs (with 'relative' necessity) to no white.

(2) If animal belongs to all men and animal belongs to all Eskimos, then man belongs to all Eskimos.

These are propositions of the form "If *A* then *B*", in which the antecedent is composed of the conjunction of the first and second component propositions (the premisses) and the consequent is the third component. Further, both propositions are true (the truth of the second and third com-

ponents of (1) is explicitly accepted in Aristotle's text), since the consequents are true. The two propositions have important formal similarities, proposition (1) has the form:

(F1) If $(x)(Ax \rightarrow \bar{B}x)$ and $(x)(Cx \rightarrow \bar{B}x)$, then $(x)(Cx \rightarrow \bar{A}x)$[20]

and proposition (2) has the form:

(F2) If $(x)(Ax \rightarrow Bx)$ and $(x)(Cx \rightarrow Bx)$, then $(x)(Cx \rightarrow Ax)$.

The universal quantifier (x) before each of the six components indicates that for all possible objects x it holds that, for example, if x is an A then x is also a B $(Ax \rightarrow Bx)$. Since in (1) and (2) concrete terms stand in the place of the variables A, B, C and the individual variables are all bound by universal quantifiers, the propositions (1) and (2) are in every respect 'saturated' functions (Frege) or 'closed' schemata (Quine) – in short, propositions which are true or false, and hence, in our case, both true. An "If...then" proposition is true, as we have said, provided only that the consequent is true in case its antecedent is true.

What, then, does it mean when Aristotle asserts as he does that the conclusion of (1), or rather the proposition which comprises the conclusion of (1), is 'relatively' necessary, or rather 'relatively' necessarily true? Does it mean that it is the consequent of a true "If...then" proposition the antecedent of which is true, and that it is thus 'necessarily' true in this sense? That would make sense; but it is demonstrably not what Aristotle wants to say when he maintains that it is a 'relatively' necessary proposition. For in *this* sense the consequent of (2) would also be a 'relatively' necessary proposition. But on his principles Aristotle must refuse it this title, since in his view the operator "relatively necessary" governs the conclusion of a valid inference *qua conclusion*, and proposition (2) is not a valid inference at all. To say that proposition (1) is a valid inference means simply that it is the result of inserting concrete terms in a propositional *schema* which is such that a true proposition results whatever terms are substituted for its variables A, B, C, that therefore no case can arise in which the conjunction of its antecedents is true and its consequent false. To say that proposition (2) is not a valid inference means simply that it is possible to find terms which when inserted into (F2), the schema from which (2) was produced, produce a false proposition. If we insert in (F2) "horse" for A, "animal" for B, and "man" for C, we reach the

false proposition:

(2*) If animal belongs to all horses and animal belongs to all men, then horse belongs to all men.

Thus that (F2) is not a law of logic means precisely that not all propositions of the form (F2) are true. And the assertion that (F1) is a law of logic (in fact the syllogism *Camestres* (II), cf. *APr. A* 5, 27a9–14) means that all propositions of the form (F1) are true. (Moreover, if *all* propositions of the form (F2) were false, then (FC2) – the schema formed from (F2) by replacing the consequent by its contradictory – would be a law of logic.)

In the case of syllogisms with concrete terms in the place of variables, 'relative' necessity cannot be taken as an operator on the conclusion. Here again it is an operator on the whole syllogism; it advertises that the argument is produced by the insertion of concrete terms in a law of logic or a schema which yields a true proposition when prefixed by universal quantifiers over the term-variables *A*, *B*, *C*. Proposition (2) cannot take the same modal operator because the schema from which it is produced by insertion of concrete terms for variables, turns into a false proposition if it is preceded by quantifiers of the same sort. For it is not true that (F2) gives a true proposition for all possible terms *A*, *B*, *C*.

If the expression for Aristotle's 'relative' necessity is a universal quantifier over the term-variables *A*, *B*, *C*, then the notion of 'relative' *possibility* which Aristotle occasionally employs corresponds to an *existential* quantifier over the same variables. And the 'relative' necessity of a syllogism would be controverted by asserting the 'relative' possibility of its negation. 'Relative' possibility is proved by producing terms whose insertion makes the premisses true propositions and the (presumptive) conclusion false. (In fact Aristotle's procedure is rather different: cf. § 31.) Since Aristotle's 'relative' necessity corresponds to a universal quantifier and his 'relative' possibility to an existential quantifier over the term-variables, it can easily be deduced that he was right in allowing 'relative' necessity to remain unexpressed in his syllogisms, but that he made a logical error when, as in the case given on page 20, he omitted the expression for 'relative' possibility. For a proposition containing free variables can be asserted if we assert, and mean to assert, its universal closure. A proposition containing variables so chosen that it only turns into a

true proposition for certain arguments cannot be asserted unless the existential quantifier is expressly added. Thus we can say, for example, "Philosophers are men", since it is true that for all x, if x is a philosopher then x is a man. But we cannot say "Men are philosophers" – or rather to say this is to say something false – since it is only for some x that it is true that x is a man and x is a philosopher. The proposition "*primum vivere, deinde philosophari*" has, we can see, a logical basis.

§ 9. The Identity of Aristotle's Two Types of Necessity

We have so far shown that Aristotle was wrong to suppose what he called 'relative' and 'absolute' necessity to be two sorts of necessity which could in certain cases belong to one and the same proposition. For in fact his so-called 'absolute' necessity can only belong to propositions of the form "*A* belongs to all *B*", which appear as components of a syllogism; whereas 'relative' necessity belongs to the syllogism itself, that is, to a proposition of the form "If *A* then *B*", – hence to a fundamentally different sort of proposition. But is the necessity, just because it belongs to fundamentally different propositions, therefore a different necessity in each case? Or can we invert Aristotle's assertion that 'relative' and 'absolute' necessity are two different types of necessity which may in certain cases belong to one and the same proposition, and say instead that Aristotle's 'relative' and 'absolute' necessity are one and the same necessity which belong to fundamentally different types of proposition?

This question will now be discussed. We have already seen that Aristotle regarded a proposition such as "Animal belongs to all men" as 'absolutely' necessary. In the section on modal syllogistic, where he uses this type of proposition, Aristotle does not discuss the question why such propositions are 'absolutely' necessary; although the notion of 'absolutely' possible propositions, or more precisely the conditions under which a predicate belongs ('absolutely') possibly to a subject are defined there at length (*APr. A* 13). That Aristotle himself felt the need of a similar discussion of 'absolutely' necessary propositions is perhaps hinted at in the sentence: περὶ μὲν οὖν τοῦ ἀναγκαίου, πῶς γίνεται καὶ τίνα διαφορὰν ἔχει πρὸς τὸ ὑπάρχον, εἴρηται **σχεδὸν** ἱκανῶς (immediately before the corresponding discussion of 'absolute' possibility, *A* 12, 32a15–16). However, later in the *Posterior Analytics*, *A* 4–8, he did treat in detail the

notion of a predicate's belonging with 'absolute' necessity to a subject. Study of the theory there propounded might at first lead one to believe that our interpretation of the operator within the syllogistic as a universal quantifier stands in need of correction, in so far as it depends on Aristotle's equation of "necessarily" with "always" (τὸ ἀεὶ ὄν with τὸ ἐξ ἀνάγκης ὄν cf. p. 27). For the propositions "All men are asleep" and "No white thing is a man", which mathematical logic can only represent by means of the universal quantifier – $(x)(\phi x \rightarrow \psi x)$ or $(x)(\phi x \rightarrow \sim \psi x)$ – are in Aristotle's sense certainly not 'absolutely' necessary like the propositions "All men are animals" or – the standard example of *APst. A* 4–8 – "All triangles have an angle-sum of 180°".

In this section Aristotle distinguishes between two types of universal judgment, which he designates by the expressions "κατὰ παντός" and "καθόλου" (l.c. 73b25 sqq.). Only those universal judgments which assert a καθόλου ὑπάρχειν are in his sense 'absolutely' necessary propositions. The defining conditions of such a judgment are, first, that its predicate belongs to *all* objects which fall under the subject term, and secondly that its predicate belongs to these objects *as such* (καθ' αὑτό), that is, in virtue of their definition. The proposition "All men are asleep", for example, or better its Aristotelian equivalent "Sleep belongs to all men", is κατὰ παντός and fulfills the first condition which all propositions must satisfy if they are to be 'absolutely' necessary. But it does not fulfill the second condition, since sleep does not belong to man καθ' αὑτό, by virtue of his definition. This difference could be put thus: the (x) in the proposition "(x) (man $x \rightarrow$ asleep x)" (For all objects x: if x is a man, then x is asleep) is not a true quantifier over the whole realm of individuals, since it is only the objects existing at a particular time (that in which all men are asleep) which make the proposition true; whereas in a *necessary* proposition the quantifier is only subject to those restrictions which the rules of logic prescribe in general terms for all quantifiers of a particular type (e.g. a *term* cannot be substituted for the *object*-variable x, since the proposition would then be not true but meaningless). However, it is always possible to remove the restriction on the range of the variables by restricting the propositional function itself.[21] In place of the proposition "For all objects x at time t: if x is a man then x is asleep", we can construct the *equivalent* proposition: "For all objects x: if x is a man and x exists at time t, then x is asleep". In the second of these two

propositions the quantifier is genuine; the restriction has been turned into an additional qualification on the antecedent of the implication "If Ax, then Bx".

Thus the proposition "All men are asleep" does not express a necessary connexion between the terms "man" and "sleep", but a necessary connexion between the new term which is formed by the logical product of the terms "man" and "existence at time t", and the term "sleep". On the other hand the 'necessary' proposition "All men are ainmals" asserts a necessary connexion between the terms "man" and "animal", since it holds between them whatever other terms may be multiplied with "man" to form a logical product.

However, there is a formal difference of some importance between the propositions "All men at time t are asleep" and "All men are animals". Even if the quantifier (x) in each case extends over the entire realm of individuals that the laws of language allow, we could still only ascertain the truth of the first proposition by observation, whereas the second proposition follows from the definitions of "man" and "animal". "Man" is defined as an animal of a particular sort, and therefore (by the law of contradiction) all men are – with logical necessity – animals. On the other hand it is not possible to define "man" and "time t" in such a way that the truth of the proposition "All men at time t are asleep" follows from the definitions. This is, I think, the reason why Aristotle twice points out that propositions which are to be in his sense universal and necessary may not contain temporal terms (*APst. A* 4, 73a28–34; cf. *A* 8, 75b21–30). Necessary propositions are in his terminology 'eternal' truths. In this respect, however, Aristotle's 'necessary' propositions of the form "A is B" and his 'relatively' necessary propositions – syllogisms – are on a level. The truth of the propositions of the first class follows from the definitions of their terms, the truth of the syllogism follows from the definitions of the logical constants a, e, i, o and of the connectives "and" and "if ... then".

In *APst. A* 4, Aristotle introduces a still more rigorous condition for 'necessary' predication which propositions such as "All men are animals" etc., which Aristotle's modal logic has taught us to be 'absolutely' necessary, do not satisfy. What belongs necessarily must not only belong κατὰ παντός and by virtue of a definition, καθ' αὑτό, it must also belong *only* to that to which it 'necessarily' belongs (ᾗ αὐτὸ ὑπάρχειν).[22] On this

definition isosceles triangles, for example, do not 'necessarily' have an angle-sum of two right angles: only triangles have. For only triangles are unique in possessing the property "having an angle-sum of 180°"; it is not the case that isosceles triangles alone have the property – they share it with equilateral, rightangled and all other triangles.

This condition could be formalised thus:

$$(x)\,(Ax \rightarrow Bx)\,\&\,(Bx \rightarrow Ax);$$

a predicate B belongs ἐξ ἀνάγκης to an object x of which Ax holds only if it is *equivalent* to A. In this sense the predicate "has an angle-sum equal to two right-angles" is a 'necessary' predicate of all triangles, for in fact the terms "triangle" and "has an angle-sum equal to two right-angles" are equivalent. This definition of necessary predication cannot, however, underlie Aristotle's usage of the term in his modal logic. For there he calls such propositions as "All animals are substances" or "All men are animals" absolutely necessary although it is not the case that the pairs of terms "animal", "substance", and "man", "animal", are equivalent.

For the question we are here concerned with, the important point is that Aristotle differentiates between propositions which are only κατὰ παντός and those which are καθόλου, and which alone can take the operator of 'absolute' necessity, by stating certain formal properties of the latter; and that the propositions which he asserts (or rather, parts of which he asserts) to be 'relatively' necessary have precisely those formal characteristics by which he defines 'absolutely' necessary propositions. For, just as the 'absolutely' necessary propositions there defined can be construed as assertions about properties of any *objects* whatever, so syllogisms can be construed as assertions about properties of any *terms* whatever. Correspondingly, just as the predicate "animal", for example, belongs to all men as such (that is, independently of whatever else may hold of those objects which are men), and therefore belongs 'necessarily' to them; so the logical properties formulated in, say, the syllogism *Barbara* (I) belong to all terms which satisfy the premisses, that is they belong independently of whatever other logical properties may belong to them.

Thus the propositions to which Aristotle attributes the operator "relatively necessary" satisfy the conditions which he introduced to define "absolutely necessary". We have thereby shown that the two types of necessity distinguished by Aristotle are one and the same, and that an

adequate expression for this necessity is the universal quantifier. In the case of 'absolute' necessity the operator binds individual-variables, in the case of 'relative' necessity term-variables. Aristotle defines 'relative' and 'absolute' necessity as two types of necessity which may in certain cases belong to one and the same proposition: in fact he is dealing with one and the same necessity applied to two fundamentally different types of propositions, in the one case, universal propositions about individuals, in the other, universal propositions about predicates or terms.

§ 10. Subject and Predicate

An Aristotelian καθόλου proposition is, in the terminology of mathematical logic, a proposition about all actual objects, since it asserts the truth of an "If ... then" proposition for all possible values of the variables x, that is, of the object variables. We may say that the 'subject' of this proposition is the universal class of individuals, and that its 'predicate' is the truth of the "If ... then" proposition for each member of this class. That this conception provides no difficulties is shown by the following considerations. The negation of the implication "$Bx \rightarrow Ax$" is the conjunction "Bx and not Ax". The proposition that for all x the implication is true, can be replaced by the assertion that *no* x belongs to the class "B and not A". Let "man" stand for B and "mortal" for A: then the proposition "All men are mortal" is patently synonymous with the proposition "There is nothing which belongs to the class of immortal men". This can in turn be transformed into the assertion that all individuals belong to the complement-class of the class of immortal men, that is the class to which everything which is not an immortal man belongs. Not being an immortal man is therefore the property which, according to this interpretation, is affirmed of any x whatever by the proposition "All men are mortal".

In mathematical logic the proposition "The A belongs to all B" has the form $(x)\,(Bx \rightarrow Ax)$. In Aristotle's view, the subject of this proposition is not the universal class of individuals but the class of individuals of which B holds, in short the class B; and the predicate of the proposition is the predicate A. Thus Aristotle's conception differs from that of mathematical logic in that he restricts the 'universe of discourse'[23] to those objects of which B holds. It follows from this restriction that, although Aristotle's

proposition is never true when that of mathematical logic is false, in certain cases it is neither true nor false – that is, it is meaningless – when on the mathematical interpretation it is true. This occurs whenever an individual is substituted for x which does *not* belong to the class B. Aristotle's proposition says nothing at all about this case; in mathematical logic, on the other hand, the proposition remains true: it asserts only that the predicate A belongs to x *if* the predicate B belongs to it. The difference between these two ways of conceiving the subject and predicate of such propositions can be illuminated by means of a diagram:

$$\underset{\mathrm{S}}{(x)}\ \underset{\mathrm{P}}{(\overset{\mathrm{S}}{Bx} \rightarrow \overset{\mathrm{P}}{Ax})}$$

The arrow below the line connects subject and predicate according to mathematical logic, that above the line according to Aristotle.

Can we discover an analogous distinction between ways of construing subject and predicate in the case of the propositions which Aristotle called 'relatively' necessary? Let us consider the form of one of these propositions, the syllogism *Barbara* (I) for example:

$$(A, B, C)\ \{[(x)\ (Bx \rightarrow Ax) \,\&\, (x)\ (Cx \rightarrow Bx)] \rightarrow (x)\ (Cx \rightarrow Ax)\}$$

Mathematical logic finds no difficulty here. The subject of the proposition is the class of all syllogistic terms, more precisely the class of all triples of syllogistic terms; and the predicate, just as in the earlier example, is the truth of an "If ... then" proposition for each member of this class of triples. It is true that the constituents of this "If ... then" proposition are formally different from those of the previous one: there we had an implication between properties of an individual, or rather an implication between propositions asserting properties of an individual: here we have an implication between implications. However, it is not difficult to see that an implication is a property of a term in a sense analogous to that in which a simple predicate is the property of an object: we are *describing* a term A when we say that it is subordinate to a term B ($Ax \rightarrow Bx$), just as we describe an object when we assert that a predicate, say "man", belongs to it.

Thus the lower arrow in our example can easily be drawn in: the subject

is the class of term-variables (A, B, C), the predicate the implication between the implications, or, more precisely, between the conjunction of two implications and the third implication.

How should we draw the upper arrow if we are to represent Aristotle's notion of subject and predicate in this case? By analogy with the proposition we analysed first the *subject*, according to Aristotle, ought to be the class of all terms which when substituted for the variables make the premisses true. Similarly, the 'universe of discourse' would here be, not the universal class of term triples, but the class of term triples which have the property set out in the antecedent. The predicate should plainly be the whole proposition "If Ax, then Cx".

$$\underset{\substack{\uparrow \\ \mathrm{S}}}{(A, B, C)}\ \{[(x)(Bx \to Ax) \overset{\substack{\mathrm{S} \\ \downarrow}}{\&} (x)(Cx \to Bx)] \underset{\substack{\uparrow \\ \mathrm{P}}}{\to} \overset{\substack{\mathrm{P} \\ \downarrow}}{(x)(Cx \to Ax)}\}$$

Aristotle states 'absolutely' necessary propositions in the form "Animal belongs necessarily to all men"; an analogous formulation of a 'relatively' necessary proposition, say *Barbara* (I), would run like this: "The property of the first term's being a subordinate term to the last term belongs necessarily to all term triples which are such that the first term is subordinate to the second and the second to the third". Aristotle did not develop this mode of speech, although it could be developed from the principles of his logic. Rather he treated as predicate of the syllogism, not the whole conclusion or the implication stated in it, but simply the predicate of the conclusion itself. He tried to express the formal difference between a proposition of the form "The A belongs to all B" and a syllogism by pointing to the fact that the predicate of the conclusion belongs to its subject with 'relative' necessity. We have shown that it is wrong to use this mode of expression. However, our further reflections have shown that Aristotle could, consistently with his principles, have taken as the predicate of a syllogism the conclusion itself and as the subject the class of term triples which satisfy the premisses.

§ 11. Maier on 'Relative' Necessity

The word "ἀνάγκη" which frequently but not invariably occurs in the

formulation of valid syllogisms in the *Prior Analytics* caused Łukasiewicz no difficulty: in the first edition of his *Aristotle's Syllogistic* his discussion occupied precisely two pages (AS, pp. 10–12); the second edition is more expansive (cf. pp. 143 sqq.): but it is still clear that the logically exact expression for this 'necessity' is the universal quantifier over the variables occurring in the 'necessary' proposition. Maier, on the other hand, makes Aristotle's distinction between 'relative' and 'absolute' necessity the culmination of his discussion. Łukasiewicz exposed his view as "a bad philosophical speculation", whose source is to be found in his failure to see the connexion between modal operators and universal quantifiers. In fact no reader of Maier's book will be satisfied with his vague and tortuous explanation of Aristotle's distinction between 'absolute' and 'relative' necessity – a distinction which according to Maier "is in perfect accord with the facts" (SdA II, 2, p. 243). The distinctions between "real-syllogistic" and "real-metaphysical" consequence (p. 242), between "ontologically valid" and "metaphysical" consequence (p. 248), and the account of 'relative' necessity as "a consequence simply of language and thought, not of being" (p. 251), seem merely to give new *names* to Aristotle's two types of necessity; the inconclusiveness which characterises the whole section is as it were epitomized in the circular proposition which ends it: the way in which Aristotle wrestles with these problems shows only "that at root he understood syllogistic consequence as corresponding to the peculiarity of the principle of inference: as logico-ontological consequence to which specifically syllogistic necessity adheres" (p. 255). What we are meant to understand by "specifically syllogistic consequence" and "peculiarity of the principle of inference", is made no clearer by such a sentence as this, from the chapter on "The Principle of Inference": the 'essence' of the syllogism is said to lie in the fact that "thanks to the synthetic power which resides in the supremacy of the ontologically universal over the particular, it attains from its premisses, with really valid consequence, a completely new conjunction or disjunction of terms" (II, 2, p. 275). However, once anyone takes his stand on Aristotle's distinction between an 'absolute' and a 'relative' necessity, he must find it difficult to keep his mind on the alleged difference between a 'real necessity' and a 'logico-ontological consequence' without taking refuge in such expressions as "synthetic power", "supremacy of the universal" and "creative power of the metaphysically universal" (p. 248).

Aristotle, however, as we have shown, never in his logical writings distinguishes between a 'logical' and a 'real' or 'metaphysical' necessity. The distinction between 'relative' and 'absolute' necessity can be worked out on the basis of Aristotle's own language entirely within the province of logic. We showed further that the distinction, as Aristotle thinks of it, is incorrect. However, the distinction he is trying to make is not a mere fiction; it can, strange though it may seem, be presented as the distinction between propositions with universal quantifiers over object-variables and propositions with universal quantifiers over term-variables – and it thus falls wholly in the field of logic. And this should win Aristotle all honour.

NOTES

1. Aristotle's stronger thesis that in certain specified cases *one* apodeictic premiss *suffices* for the inference of an apodeictic conclusion was, as is well known, attacked by Theophrastus and Eudemus. In opposition to this they held fast, even in modal logic, to the principle later formulated by the Schoolmen as "conclusio sequitur partem debiliorem." On this cf. Bocheński, FL, pp. 94–101, 116 sqq.; HFL, pp. 81–88; 101–103. Aristotle's position is tenable on the assumption that the modal operator determines not the whole proposition but only the predicate. Whether or not Aristotle did read the operator in this way is an interpretative crux – the resolution of which is not relevant to the statement in the text.
2. On this cf. Frege's remark (*Begriffsschrift*, 1879, p. 4): "Apodeictic judgments differ from assertoric in that the former indicate the existence of universal judgments from which the proposition can be inferred, whereas the latter do not. If I call a proposition necessary, I am giving a hint of the grounds for my judgment."
3. *A* 11, 31b16–18; b36: εἰ οὖν ἀνάγκη τὸ *B* παντὶ τῷ *Γ* ὑπάρχειν, τὸ δὲ *A* ὑπὸ τὸ *Γ* ἐστίν, ἀνάγκη τὸ *B* τινὶ τῷ *A* ὑπάρχειν ... εἰ γὰρ τὸ *A* τῷ *Γ* μηδενὶ ἐνδέχεται, τὸ δὲ *B* τινὶ τῷ *Γ* ὑπάρχει, τὸ *A* τινὶ τῷ *Γ* ἀνάγκη μὴ ὑπάρχειν. In both cases the 'relative' necessity of the conclusion is unexpressed: cf. below, n. 5.
4. Cf. Kühner-Gerth, *Griechische Grammatik* I, pp. 173 sqq.
5. 31b16–18 and b36 are, as we have said, special cases. Here the expression usually used for 'relative' necessity serves to express the 'absolute' necessity of the propositions which are deducible as conclusions.
6. οὐκ ἔσται συλλογισμὸς τῶν ἄκρων· οὐδὲν γὰρ ἀναγκαῖον συμβαίνει τῷ ταῦτα εἶναι (*A* 4, 26a4–5).
7. καὶ γὰρ παντὶ καὶ μηδενὶ ἐνδέχεται τὸ πρῶτον τῷ ἐσχάτῳ ὑπάρχειν, ὥστε οὔτε τὸ κατὰ μέρος οὔτε τὸ καθόλου γίνεται ἀναγκαῖον· μηδενὸς δὲ ὄντος ἀναγκαίου διὰ τούτων οὐκ ἔσται συλλογισμός (*A* 4, 26a5–8).
8. Similar passages: *A* 13, 32b29–30; *A* 15, 34b5–6 (secl. Becker); *A* 28, 44b27–28; 45a9 (secl. Ross).
9. "συμβαίνει" here has the singular meaning – noticed by Ross, APPA, p. 359, but not recorded in Bonitz' *Index*: – "It *may* happen".
10. A further example is *APr. A* 4, 26b5–6.: τούτῳ καὶ παντὶ καὶ οὐδενὶ ἀκολουθήσει τὸ πρῶτον, where the sense requires ἐνδέχεται ἀκολουθεῖν. Similarly *A* 17,

37a20–24, where in line 22 παντὶ γὰρ ὑπάρχει must mean παντὶ γὰρ ἐνδέχεται ὑπάρχειν, cf. Ross, APPA, p. 353.

11. *APst. A* 4, 73a21–74a3. On this cf. § 9.
12. *APr. A* 11, 31b8-10.
13. *De Int.* 9, esp. 18b9–19b4. On this cf. G.E.M. Anscombe, 'Aristotle and the Sea Battle', *Mind* **65** (1956), 1–17; Bocheński, FL, pp. 73 sqq.; HFL, pp. 62–3; R.J. Butler, 'Aristotle's sea fight and three-valued logic', *Philosophical Review* **64**, (1955), 264–274 and Butler's reference to the relevant discussion between D.C. Williams and L. Linsky.
14. οὐχ ὑπάρξει δὴ οὐδ' ὁ ἄνθρωπος οὐδενὶ λευκῷ, ἀλλ' οὐκ ἐξ ἀνάγκης· ἐνδέχεται γὰρ ἄνθρωπον γενέσθαι λευκόν, οὐ μέντοι ἕως ἂν ζῷον μηδενὶ λευκῷ ὑπάρχῃ· ὥστε τούτων μὲν ὄντων ἀναγκαῖον ἔσται τὸ συμπέρασμα, ἁπλῶς δ' οὐκ ἀναγκαῖον. (*APr. A* 10, 30b36–40).
15. And, moreover, in its primary meaning: *Met. Δ* 5, 1015a33–36.
16. Cf.: *Met. E* 2, 1026b28–29: ἐπεὶ οὖν ἐστὶν ἐν τοῖς οὖσι τὰ μὲν αἰεὶ ὡσαύτως ἔχοντα καὶ ἐξ ἀνάγκης, οὐ τῆς κατὰ τὸ βίαιον λεγομένης ... (cf. *Met. Δ* 5, 1015a33–b9); *Phys. B* 5, 196b12: οὔτε τοῦ ἐξ ἀνάγκης καὶ αἰεὶ οὔτε τοῦ ὡς ἐπὶ τὸ πολύ; *Met. K* 8, 1064b32–34: πᾶν δή φαμεν εἶναι τὸ μὲν αἰεὶ καὶ ἐξ ἀνάγκης (ἀνάγκης δ' οὐ τῆς κατὰ τὸ βίαιον λεγομένης, ἀλλ' ᾗ χρώμεθα ἐν τοῖς κατὰ τὰς ἀποδείξεις) ...; 1065a1–3: ἔστι δὴ τὸ συμβεβηκὸς ὃ γίγνεται μὲν, οὐκ αἰεὶ δ' οὐδ' ἐξ ἀνάγκης οὐδ' ὡς ἐπὶ τὸ πολύ ... . Cf. K.J.J. Hintikka, 'Necessity, Universality, and Time in Aristotle', *Ajatus* **20** (1957), 65–90.
17. Aristotle himself uses the formula "If *A*, then *B*" (in which *A* stands for the two premisses and *B* for the conclusion) to express a syllogism: e.g. *APr. A* 15, 34a22–24; *B* 2, 53b12–24.
18. Cf. Whitehead and Russell, PM I, p. 5; W.V.O. Quine, *Methods of Logic*, 1953, p. 97.
19. Those acquainted with the rudiments of mathematical logic will recognize that this formal distinction between the syllogism *Barbara* (I) and the proposition with a universal quantifier over the term variables in both the antecedent and the consequent corresponds to the formal distinction between (1) $(x)(\phi x \rightarrow \psi x)$ and (2) $(x)(\phi x) \rightarrow (x)(\psi x)$. The first expression entails the second, but not vice versa. The relation between (1) and (2) becomes particularly clear if we adopt the legitimate device of limiting the range of objects which can be substituted for x (the argument range of x) to a finite set of individuals (cf. Quine, *Methods of Logic*, p. 88). Let the range of x be the set of examination papers in a definite subject which are marked in the course of one academic year. If a father promises his son a reward for each paper which is graded "good" or better, his promise has the form (1); a proposition of the form (2) would promise to reward each individual paper provided that all of them turned out to be good. The son would justifiably be more pleased with the first promise than with the second: the first gives him the prospect of a reward even if only one of his papers gains the required mark, and it clearly includes the second as a limiting case.
20. "$\bar{A}$" must be read "not-*A*"; it denotes the predicate contradictory to *A*.
21. Cf. Quine, *Methods of Logic*, p. 184.
22. *A* 4, 73b26–28: καθόλου δὲ λέγω ὃ ἂν κατὰ παντός τε ὑπάρχῃ καὶ καθ' αὑτὸ καὶ ᾗ αὐτό. φανερὸν ἄρα ὅτι ὅσα καθόλου, ἐξ ἀνάγκης ὑπάρχει τοῖς πράγμασιν.
23. The expression is de Morgan's (*Formal Logic*, 1847); its fitness has given it wide currency.

CHAPTER III

PERFECTION

§ 12. 'Perfect' and 'Imperfect' Syllogisms

It is a familiar and much-discussed feature of Aristotle's syllogistic, that in it the distinction between 'perfect' and 'imperfect' syllogisms plays an essential part. This distinction divides all valid syllogisms into two classes, the second of which is 'reduced' by means of certain logical operations to the first class and thereby proved, while the first class is taken as axiomatic and assumed without proof. Up to now the nature of this distinction has not been properly understood. The ancients debated whether or not Aristotle recognised the *validity* of the imperfect inferences, and whether, if he recognised them as valid, he had the right to call them 'imperfect'.[1] The 19th century historians of philosophy, who equated the Aristotelian with the traditional syllogism, were no longer able to see the formal peculiarities of Aristotle's 'perfect' inferences in the traditional formulation – in which as a matter of fact they disappear –, and were obliged to seek other grounds for the dichotomy. The fact that all the inferences which Aristotle calls 'perfect' belong to the first figure encouraged them to speak of perfect *figures* rather than of perfect *syllogisms*, and to suppose that perfect syllogisms were perfect just because they belonged to the perfect, that is to say the first, figure. Aristotle's reasons for calling this figure the first (it yields conclusions in all forms, *a*, *e*, *i*, and *o*, whereas the second figure does not allow *a* or *i* and the third does not allow *a* or *e* as conclusions (*A* 4, 26b23–33)), and his assertion in the *Posterior Analytics* (*A* 14, 79a17–32) that the first figure is the truly 'scientific' figure, were taken as reasons for the 'perfection' of this figure and hence of the syllogisms contained in it. From here it was no long step to the opinion which remains widespread today and is supported by the authority of Prantl, Zeller, Überweg, Maier and Trendelenburg, that Aristotle's logic, in particular his syllogistic, stands in such intimate connexion with his ontology and 'conceptual metaphysics' that it cannot be understood apart from this metaphysical background.[2] The logical dis-

tinction between 'perfect' and 'imperfect' inferences cannot be grasped, it was said, unless we accept the theory that the first figure is 'the perfect figure' just because in it alone, and in no other figure, the middle term can be the metaphysical cause of the fact expressed in the conclusion.

This whole theory, which is so firmly embraced by Carl Prantl in his *Geschichte der Logik im Abendlande* that he regards a logic without this metaphysical background as an empty game (Prantl I, p. 348), and which governs Kant's essay of 1762, *Die falsche Spitzfindigkeit der vier syllogistischen Figuren*, has no support in the text of the *Analytics*: rather, the text palpably proves it false. First, in Aristotle's view, not all syllogisms which belong to the first figure *are* 'perfect': but this would have to be the case if an argument were 'perfect' just because it belonged to the ('perfect') first figure. Secondly, Aristotle never calls the first figure τὸ τέλειον σχῆμα, even though he says that this figure is, for the reasons given, particularly important and rightly called the *first* figure. Thirdly, he brings forward as a *justification* of the first figure's claim to its privileged position the fact that all its (assertoric) syllogisms are perfect (*APr. A* 4, 26b28-33). This argument would be circular if it were the case that the so-called 'perfect' syllogisms were 'perfect' just because they belonged to the first figure. The precedence of the first figure, which Aristotle himself recognised, and the 'perfection' of the majority of the arguments in it are therefore, in Aristotle's view, two quite different matters; and if they are related it is not that the precedence of the figure causes the 'perfection' of the syllogisms, but the other way about: the fact that 'perfect' syllogisms occur only in the first figure ensures, among other virtues, the precedence of the figure.

Moreover, the distinction between 'perfect' and 'imperfect' inferences which Aristotle introduces is quite clearly defined in the first chapter of the *Prior Analytics*, immediately following his famous definition of the expression "syllogism". I quote the passage in translation: "A syllogism is a proposition in which, certain facts being stated, something different from these facts follows with necessity (simply) from their being so. By "from their being so" I mean the same as "following because of them", and by "following because of them" I mean that no term outside those given is needed in order that the necessity should occur. I call perfect a syllogism in which nothing else (apart from what is given) is needed for the necessity to appear, imperfect a syllogism which needs one or more

things for that purpose, things which are indeed necessary by reason of the given terms but which are not explicitly stated through the premisses" (*A* 1, 24b18–26)[3].

These lines present a host of difficulties, due in part to a somewhat careless but readily corrigible mode of expression, and in part to Aristotle's general reluctance, here particularly in evidence, to commit himself to more than the matter under immediate consideration requires. First, the definition of the syllogism is obviously too wide. It is satisfied by every logically true assertion of an implication between two propositions in which the antecedent is different from the consequent. But in Aristotle's view only those logically true implications are 'syllogisms' in which the antecedents and the consequent are propositions of the form *AaC*, *AeC*, *AiC*, or *AoC*. Again, the expression "διὰ ταῦτα συμβαίνειν" is not, as Aristotle asserts, equivalent to the expression "τῷ ταῦτα εἶναι" but at best to "τῷ ταῦτα εἶναι συμβαίνειν"; and the phrase "μηδενὸς ἔξωθεν ὅρου προσδεῖν πρὸς τὸ γενέσθαι τὸ ἀναγκαῖον" does not, as Aristotle asserts, elucidate "διὰ ταῦτα συμβαίνειν", but rather the expression which is in fact employed in the definition of the syllogism, "διὰ ταῦτα ἐξ ἀνάγκης συμβαίνειν". Further, "πρὸς τὸ γενέσθαι τὸ ἀναγκαῖον" is ambiguous: it can mean "in order for the necessary to come into being", and also "in order for necessity to come into being", and only this latter alternative appears to express Aristotle's intention. Finally, in these definitions of the syllogism and of the perfect syllogism Aristotle again uses the concept of 'relative' necessity[4], the dubious nature of which has been exposed in the preceding section.

However, these verbal infelicities do not affect our understanding of the difference between 'perfect' and 'imperfect' arguments: a perfect argument is an argument in which the defined necessity not only occurs but '*appears*' or is transparent[5], whereas an imperfect argument, although it possesses this necessity, must undergo certain operations before its necessity 'appears' or is transparent. In a word, perfect syllogisms are *self-evident* syllogisms. Thus Aristotle calls the 'perfect' arguments *clear* arguments, φανερὸς συλλογισμός[6]. There is no doubt that the syllogisms Aristotle calls 'imperfect' are valid, and that Aristotle thought they were. The ancient commentators were guilty of supposing that Aristotle denied the validity of imperfect syllogisms; they were followed by Prantl who, in his *Geschichte der Logik im Abendlande*, offered it as his and Aristotle's

opinion that imperfect syllogisms "have no probative force" (I, p. 374 cf. p. 271); Kant had made the same assertion in § 5 of his essay of 1762 *Die falsche Spitzfindigkeit der vier syllogistischen Figuren*. The main source of this misunderstanding appears to be Aristotle's habit of sometimes calling imperfect syllogisms potential syllogisms (δυνατὸς συλλογισμός: e.g. *A* 5, 27a2; *A* 6, 28a16; *A* 24, 41b33). This usage cannot indeed fail to suggest to anyone acquainted with Aristotelian terminology the conjecture that such inferences are not yet proper syllogisms, and hence not properly valid. However, the context of these passages makes it quite clear that what Aristotle calls δυνατὸς συλλογισμός is not a *logos* which is potentially a syllogism (δυνάμει συλλογισμός), but a *syllogism* which is potentially a '*perfect*' syllogism (δυνάμει τέλειος συλλογισμός). From the very definition of 'imperfect' syllogisms (*A* 1, 24b24–26) it follows that they *can* ("by one or more operations") be changed into 'perfect' syllogisms; had Aristotle here defined 'imperfect' arguments as those which are *not* perfect, it would not of course follow that an 'imperfect' syllogism is "potentially a 'perfect' syllogism". Aristotle's discovery that the syllogisms in the other figures can be reduced to those in the first is thus already presupposed in his definition.

§ 13. Perfection and Evidence

The distinction between 'perfect' and 'imperfect' inferences is not one of logical validity, but one of evidence. The modern commentators[7] are agreed that Aristotle believed the syllogisms which he called perfect to be evident, and that he called them perfect for this very reason. It is strange, however, that none of them has asked the question: what justification did Aristotle have for asserting some syllogisms to be evident and others not? Ross sees in this assertion simply a reflection of the historical fact, or rather the historical guess, that Plato's dialogues contain embryonic formulations of certain syllogistic laws, which chance to answer to *Barbara* (I) and *Celarent* (I) and that Aristotle, influenced by this idea of Plato's, therefore gave pride of place to the first figure and called only first figure syllogisms 'perfect'.[8] Łukasiewicz relates Aristotle's talk of self-evidence to his realization (*APst. A* 2, 72a25–23, 73a6) that in every deductive system certain *axioms* must be presupposed without proof; for, he notes, Aristotle does in fact use the first figure syllogisms as axioms

of his deductive system of syllogistic. This is correct; however, Aristotle himself shows that it is equally possible to assume the syllogisms of the second or the third figure as axioms and then to deduce the remaining syllogisms from them (*APr. A* 45, 50b5–51a2). The evidence of a syllogism cannot therefore be the same thing as its ability to serve as axiom in the proof of other syllogisms. The question, what right Aristotle had to call the 'perfect' syllogisms self-evident, is thus as open as before; and, in view of the vital role that the distinction between 'perfect' and 'imperfect' syllogisms plays in Aristotle's syllogistic, its solution is indispensable to a correct interpretation of that theory.

First, "self-evidence" is not a logical term. That does not mean that it is senseless to use it in certain logical contexts. We can, for example, speak in mathematics of the elegance of a proof, although "elegance" is not a mathematical term. However, it is possible to turn the expression "self-evidence" into a term of logic by a necessarily arbitrary definition – for example by stipulating that a logical law shall count as "self-evident" if it can be gained by substitution in a formula of the propositional calculus, or if it belongs to a fixed group of n logical laws. Similarly, we could turn "elegance" into a mathematical term by decreeing that every proof shall be "elegant" which uses the technique of mathematical induction, or the 'diagonal' method, etc. Thus Aristotle could have defined 'perfect' (and self-evident) arguments by the simple means of stipulating that the assertoric syllogisms of the first figure were to be *called* self-evident. However, this is clearly not his method: he is making an assertion and not proposing a definition when he calls these syllogisms 'perfect' *because* they are self-evident: he is asserting *that* they are self-evident.

It is absurd to say that a particular logical law is valid for some men and not for others. But is it not only not absurd, but often true, to say that a logical proposition is evident to some and not evident to others. Indeed I can even say that a proposition is evident to me today but was not evident before. Now when Aristotle says that the 'perfect' arguments are self-evident, he cannot mean that they are evident *to him*. His words do not indeed assert that such inferences are evident to everyone, but they certainly do mean that they are evident to every adult, normally endowed and sufficiently interested reader. The right to make such an assertion can only stem either from a general enquiry which has given the information that such arguments are as a matter of fact evident to

such persons, or else from the realisation that the inferences in question have certain well-defined formal characteristics which do not belong to any of the 'imperfect' inferences and which justify the assumption that a normally endowed man would be convinced of the truth of the first class of syllogisms *substantially more easily* than he would be of the truth of the second class. Since Aristotle tells us nothing of an empirical justification for his assertion about the self-evidence of these syllogisms, we must find out (a) what syllogisms Aristotle calls 'perfect', (b) what formal properties belong to all and only those syllogisms which Aristotle calls 'perfect', and (c) whether these properties in fact justify the assumption that the validity of a 'perfect' syllogism can be seen substantially more easily than that of a syllogism which does not possess those formal properties.

In replacing "self-evident" here and earlier by the expression "substantially easier to see", I am trying as far as possible to do justice to two things, Aristotle's own terminology and everyday usage. For Aristotle, 'perfection' is not an attribute which a syllogism can have in greater or less degree, and consequently the self-evidence in which the perfection of an inference consists must not belong to it in greater or less degree; it is either present or absent.[9] However, it is linguistically quite legitimate to call one proposition more or less evident than another. It is an essential feature of Aristotle's terminology that 'imperfect' arguments are not only not so immediately transparent as 'perfect' ones, but that, although they are valid, they are not transparently valid at all and can only become so by the application of those logical operations by means of which a 'perfect' syllogism can be produced from an 'imperfect' one – for example, conversion or reductio ad absurdum.[10] Thus, while it is meaningless, in Aristotle's view to talk of degrees of self-evidence or of perfection, it is not so to talk of degrees of imperfection: for a syllogism which can be transformed into a perfect inference by just *one* operation (ἑνός, *APr. A* 1, 24b25) is much less imperfect than one which needs more operations (πλειόνων, ib.) to become transparent. Clearly Aristotle's efforts to divide all syllogisms into two classes of evident and non-evident arguments drew him away from ordinary usage. However, since every syllogism has a position in a graduated scale of evidence, a dichotomy of this sort only escapes the charge of being purely arbitrary if it can be shown that within the scale one point is particularly striking, has particular importance,

that therefore the syllogisms above this point are substantially more perfect than those below it. Thus the expression "substantially easier to see" witnesses at once the plain fact that evidence is a matter of degree and also our recognition that Aristotle would be justified in calling some syllogisms simply evident and the others simply non-evident, provided that he made the break at a point where the degree of evidence takes a striking leap upwards.

§ 14. Perfect Assertoric Syllogisms

Terminological note· From § 14 to the end of the book, for reasons which will be apparent from the text itself, I shall give the vowels we discussed in § 1 (*a*, *e*, *i*, and *o*) a *new* meaning different from that of traditional logic. This is necessary because from now on the *traditional* symbolism is to be used to help represent *Aristotelian* syllogisms which, as was shown in § 4, are formulated differently from traditional syllogisms. In his formulations of syllogistic propositions, instead of the copula ("All/some ... are/are not ..."), Aristotle uses the expressions "... belongs/does not belong to all/some ..." or "... is said/is not said of all/some ...". In Aristotle, the predicate usually stands at the beginning of the proposition and the subject at the end; in traditional logic the reverse holds. The logical relation between A and B of course remains the same, whether we express it, with Aristotle, by "A belongs to all B", or, with traditional logic, by "All B is A" just as the relation of magnitude between two numbers x and y remains the same whether we express it by "x is greater than y" or by "y is smaller than x". The change in question consists in this: in the following pages the formula "AaB" will be used to represent the Aristotelian proposition "A belongs to all B", and not, as is customary elsewhere, the traditional proposition "All A is B": similarly for *e*, *i*, and *o*. "AaB" in the *new* sense of "*a*" corresponds to "BaA" in the *traditional* sense of "*a*". Aristotle looks at the logical relation of the terms from the point of view of the predicate, traditional logic from that of the subject. Both assert the same relation, but from different directions. In modern terminology, "All ... are ... " and "... belongs to all ..." would be described as *converse* relations (details will follow). Thus in what follows the vowels *a*, *e*, *i*, and *o*, serve to symbolise the relations which are the *converse* of those they symbolise in *traditional* logic. It might be thought

best to symbolise the Aristotelian schemata simply by affixing the usual symbol for conversion in modern logic (a tilde over the sign for the relation) to the traditional symbols, and thus to write, for example, "$A\tilde{a}B$" for "A belongs to all B". However, this might easily result in confusion, since in the following discussion the Aristotelian propositions must themselves be again converted; for this reason it is preferable to change the meaning of the traditional symbols in the way we have proposed.[11] (*End of note.*)

Which arguments does Aristotle call perfect? With certain important and instructive exceptions, all syllogisms of the so-called first figure and them alone. The formal properties of the first figure are therefore a necessary but not a sufficient condition for the perfection of a syllogism. *Ceteris paribus* a syllogism of the first figure must thus be substantially more evident than a syllogism of the second or third figure. Has this assertion of Aristotle's any straight-forward sense? that is, has it a sense which does not presuppose the understanding of certain historical facts or extra-logical (e.g. ontological or epistemological) theories? Let us take the syllogism *Barbara* (I), in its Aristotelian formulation: "If the A belongs to all B and the B belongs to all C, then A must belong to all C", or, with concrete terms: "If animal belongs to all men and man belongs to all Greeks, then animal must belong to all Greeks". Let us compare this syllogism with *Darapti* (III): "If the A belongs to all B and the C belongs to all B, then the A must belong to some C", or, with concrete terms: "If mammal belongs to all whales and aquatic animal belongs to all whales, then mammal must belong to some aquatic animals". No one would deny that *Barbara* is more transparent than *Darapti*. But we might doubt wherein its transparency lies. For example, we might guess the reason to be that *Barbara* contains three (formally identical) *a*-propositions and is the only syllogism to have this characteristic, whereas the conclusion of *Darapti* is formally different from its premisses. The *cetera* would then not be, as we required, *paria*. We would therefore have to compare, for example, *Celarent* (I) and *Cesare* (II), *Darii* (I) and *Datisi* (III), *Ferio* (I) and *Festino* (II) or *Ferison* (III). Let us restrict ourselves, for the sake of economy, to one pair, say *Darii–Datisi*. The *ceteris paribus* clause is strongly fulfilled in this case: in fact the two syllogisms differ only in belonging to different figures. Is: "If A belongs to all B and B belongs to some C, then A must belong to some C" substantially more

transparent than "If A belongs to all B and C belongs to some B, then A must belong to some C"? We shall agree with Aristotle that it is. And the reason for the greater evidence of *Darii* can only lie in the formal properties which *Darii* exhibits as a member of the *first* figure. In the first figure alone do the two terms which form subject and predicate of the conclusion stand (in Aristotle's formulation) at either end of the compound proposition which forms the antecedent of the syllogistic implication; and the so-called 'middle' term stands in the middle in such a way as to bind together the two premisses. (Let us note here – we shall have later to give a detailed explanation – that this fact robs the expression "middle term" of all its mystery. It is the term which in the first figure – in Aristotle's formulation – stands in the middle in the manner stated; its *extension* or its 'power of mediation' are perfectly irrelevant.)

The greater evidence of the first figure syllogisms clearly depends on the position of their terms relative to one another. This alone enables us to observe, best of all in the case of *Barbara*, the transitivity of the relation "be said of". It is supremely evident that an a-step from A to B and an a-step from B to C justify our making an a-step straight from A to C; just as it is evident that an ancestor of my ancestor is also my ancestor, or that a box is in a room if it is in a cupboard which is in the room. The premisses of this (unaristotelian) syllogism would be "The box is in the cupboard" and "The cupboard is in the room" and the inference would run: "If the box is in the cupboard and the cupboard is in the room, then the box is in the room". We say in logic that relations such as "being an ancestor of" and "being contained in" in the examples quoted are *transitive*. That is, if they hold between x and y and between y and z, they must hold also between x and z. Other examples of transitive relations are equality, and our "belong to all" or "be said of all". Not all relations are transitive: friendship, for example, is not: if x is the friend of y and y is the friend of z it does not follow that x is the friend of z. (Of course it is not *excluded* either as it is in the case of a relation such as "father of".) Like friendship, the relations "belong to some" or "be said of some" are nontransitive. We can now easily explain why Aristotle was right to call the syllogism *Barbara* substantially more transparent (and therefore perfect), when compared to arguments in the other figures: in the formulation of *Barbara* the logical fact on which its validity depends, namely the transitivity of the relation "belong to all" between

the terms which satisfy the syllogistic assumptions, becomes supremely clear. From A we proceed to B and from B we move on to C, and it is then patent that we can also take the same step directly from A to C.

However, the other syllogisms of the first figure contain not only the relation a ("be said of all" or "belong to all") but also the relations e ("belong to no", "be denied of all"), i ("belong to some", "be said of some"), and o ("not belong to some", "be denied of some"). These relations between terms are not transitive; therefore their alleged evidence cannot lie in the transparency of a (non-existent) transitivity. Then where does it lie? This question too can be readily answered if we call a theorem of the logic of relations to our aid. *Darii*, for example, is more evident than *Datisi* (III) because in *Darii* the end of the a-step from A to B and the beginning of the i-step from B to C coincide. The two steps of the premisses, we can say, follow one another without a break. This is not the case with *Datisi*.

To make the difference between perfect and imperfect syllogisms clear and distinct we shall use certain analogies between syllogistic and a part of modern relational logic. The symbols a, e, i, and o can be interpreted as signs for *relations* between terms; thus syllogistic becomes a special case of the logic of dyadic relations, which modern logicians, notably C.S. Peirce (1839–1914), E. Schröder (1841–1902) and Bertrand Russell (*Principia Mathematica* 1910–13) have reduced to a formal calculus. Syllogistic, looked at in this way, seeks to determine in what cases the relative product of two dyadic relations (a, e, i, o) between terms (A, B, $C, \ldots$) has itself the value a, e, i, or o. This abstract mode of expression may be clarified by an example: words for kinship, such as "brother", "cousin", "father", "ancestor", "spouse", "child" etc., likewise express dyadic relations – of course between persons, not terms. That these relations hold 'between' the persons who are akin is expressed formally by placing the symbol R between the symbols for the two individuals which serve as arguments of the relation: "xRy" is to be read as "x has the relation R to y", or more simply "x is R of y", e.g. "x is father of y".

The relative product of the relations R and S (written "$R|S$") holds between the arguments x and z, if there is a y such that x is an R of y and y is an S of z. This sentence can easily be translated into, say, Russell's symbolism:

$$xR|Sz = (\exists y)\,(xRy \,\&\, ySz)$$ [12]

where $(\exists y)$ – the so-called existential quantifier – is to be read "there is a y such that ...". If in our example we insert "husband" for the relation R and "daughter" for the relation S, then the expression "x husband| daughter z" means, according to our formula, that there is a person y such that x is husband of y and y is herself daughter of z: x is the husband of a daughter of z. In this case the relative product has an existing name of its own: it is the same as the relation "son-in-law". That is, if there is a person whose husband is x and who is the daughter of z, then we can deduce logically that x is son-in-law of z – of course it remains open whether z is the mother- or father-in-law of x.

Let us now apply this to syllogistic: we let a, e, i, and o appear as possible values of the relation symbols R, S, T, U, and A, B, C, ... as values of the argument variables x, y, z, ... The premisses of *Celarent* (I) – $AeB \& BaC \rightarrow AeC$ – could then be written in the notation of relational logic as $Ae|aC$. The question what value this relative product has, i.e. to which relation it is equivalent, we could think of as parallel to the question whether the relative product "x husband|daughter z" corresponds to an already known relation. Our equivalence allows us to analyse this last expression into the two propositions "$xRy \& yRz$", in our case "x husband y & y daughter z", from which the proposition "x son-in-law z" follows as cogently as "AeC" from "$AeB \& BaC$".

By contrast, a syllogism which does not belong to the first figure, for example *Datisi* (III) – $AaB \& CiB \rightarrow AiC$ – cannot without more ado be translated into the schema of dyadic relational logic; for the logic of relations favours the ordering of the arguments of the relative product which we have already met in Aristotle's first figure: referent (the argument which *has* the relation to the other argument) and relatum (the argument *to which* the other argument is related) of the relative product are respectively referent and relatum of the simple relations from which the relative product is constructed, and the relation in which x is the referent stands in the first place, that in which z is the relatum in the second. *Datisi*, however, written as a relative product, would have the form "$xRy \& zSy$"; and this expression does not fulfill the requirements – it is, to use an expression of Lorenzen's, not '*normed*' (in German: 'normiert'). Of course, all non-normed relative products can be *transformed* into normed products, by replacing where necessary a relation by its *converse*. The converse of a relation R is symbolised by a tilde over the R,

thus: $\tilde{R}$, and it is a law of logic that whenever "xRy" holds then so too does "$y\tilde{R}x$". If R is the relation "husband", $\tilde{R}$ is "wife"; if R is "bigger", $\tilde{R}$ is "smaller" etc. The non-normed relative product which we gave can be transformed into the equivalent but normed form "$xRy \& y\tilde{S}z$". Thus it would be possible to achieve greater clarity in syllogistic and make do with only one figure, preferably the first, by adding to *a*, *e*, *i*, and *o* their converses $\tilde{a}$, $\tilde{e}$, $\tilde{i}$, and $\tilde{o}$.[13] Aristotle, however, did not admit the converses of *a*, *e*, *i*, and *o* into his formal system and he expressed the conversion of a proposition, not by converting the relational constant, but by reversing the arguments: for example, he converts "AeB" not to "$A\tilde{e}B$" but to "BeA".

There is no doubt that modern relational logic prefers the normed order of the arguments of a relative product for the very reasons which led Aristotle to call 'perfect' the syllogisms of the first figure which have analogous formal properties: when the arguments of the two component propositions (premisses) and the components themselves are put in this order, it becomes substantially easier to test the value of the relative product. It is as a matter of fact easier to see that x must be the grandfather of z if we are told that x is father of y and y father of z, than if we are told that x is father of y and z is son of y. In exactly the same way *Darii* (I) is more evident than *Datisi* (III).

It might be supposed that this formal advantage of first figure syllogisms deserves no consideration, because, with the help of relational conversion, it can always be made to *disappear*. The order of the terms "$xR_1y \& yR_2z \rightarrow xR_3z$", which we have so far taken as the mark of the first figure and which logic calls normed, can, as we have said, be maintained in the other figures too, if the syllogistic relations are replaced where necessary by their *converses*. Just as, instead of "x is greater than y" we can say "y is smaller than x", where "smaller than" is the converse of "greater than"; so in *Datisi* instead of CiB ("C belongs to some B") we could use the relation $\tilde{i}$ and write $B\tilde{i}C$ (read: "some B are C"). Then *Datisi* would run: $AaB \& B\tilde{i}C \rightarrow AiC$. However, the formal difference between *Darii* and *Datisi* is not destroyed by this artifice but merely displaced. For the difference in degree of evidence between the two syllogisms no longer depends on the position of their terms but on the fact that in *Datisi* one converse and two non-converse relations appear and it must be more difficult to assess the value of the product of such *hetero-*

geneous relations than of homogeneous ones. In fact if we adopt this notation (which has the advantage that the arguments of the relations do not need to be written out since their order remains constant) it is only the first figure that is wholly homogeneous as to its relations. In the second figure the relation of the first premiss is converse, in the third that of the second premiss and in the fourth those of both propositions. This does not mean that the fourth figure is as 'homogeneous' as the first. For the conclusion of a syllogism is always non-converse: the question of the validity of an inference is posed by Aristotle in such a way that a valid syllogism must always have a conclusion in *a*, *e*, *i*, or *o* (cf. p. 52). This convention has no consequences for $\tilde{e}$- and $\tilde{i}$-conclusions, since the relations $\tilde{e}$ and $\tilde{i}$ logically *entail* their converses – they are, to use the traditional vocabulary. 'convertible', or as Aristotle says ἀντιστρέφουσιν.[14] Such relations are called *symmetrical* in the logic of relations. $\tilde{a}$-conclusions are of no interest to Aristotle, since the only argument with an $\tilde{a}$-conclusion (a converted *a*-proposition) occurs in the fourth figure which Aristotle did not treat. It would be the syllogism: "*B* belongs to all *A* & *C* belongs to all *B* → *C* belongs to all *A*". To conform with Aristotle's convention, traditional logic changes the conclusion by virtue of the logical law "$AaB \rightarrow BiA$" (cf. pp. 138 sqq.) to "*A* belongs to some *C*", so that we have the traditional mood *Bamalip* (IV). (Aristotle knew this mood, but derived it from *Barbara* by conversion of the conclusion (*APr.* *B* 1, 53a9–12).)

The matter is different in cases where from given premisses a conclusion of the form *CoA* ($= A\tilde{o}C$) would follow. Such pairs of premisses are treated by Aristotle in *A* 4–6 as *inconcludent*. His method of proving that a pair of syllogistic propositions yields no conclusion cannot establish that the given propositions do not yield a conclusion of the form *CoA* (cf. § 22, pp. 93 sqq.). Such cases are in fact frequent: in the *first* figure the pairs *AaB* & *BeC* and *AiB* & *BeC* each yield *CoA*; the same conclusion follows from *BiA* & *BeC* and from *BoA* & *BaC* in the *second* figure, and from *AiB* & *CeB*, *AaB* & *CeB* and *AaB* & *CoB* in the *third*.

The passage in which Aristotle investigates syllogisms of this sort, which do not fit into the system of *A* 4–6, shows in a most surprising way that he must have grasped intuitively a further law of modern relational logic. I mean the following law:

If the relative product $R|S$ has the value T, then $\tilde{S}|\tilde{R}$ has the value $\tilde{T}$

(cf. Whitehead and Russell, PM I, 256: "The converse of a relative product is obtained by turning each factor into its converse and reversing the order of the factors", 34.2). An example: "uncle" (T) is the relative product of "brother" and "father" (R and S). For it is the case that $R|S \subseteq T$;[15] that is, if there is a y such that x is brother of y and y is father of z, then x is uncle of z. It is also true that $\breve{S}|\breve{R} \subseteq \breve{T}$. $\breve{S}$ in our case is "son or daughter", i.e. "child"; $\breve{R}$ is "brother or sister". And it is in fact the case that if there is a y such that x is a child of y and y is brother or sister of z then x is nephew or niece of z. "Nephew or niece" is the converse relation of "uncle". Only if Aristotle grasped these relationships can we explain why in *A* 7, 29a19–27, where he says that some of the pairs of premisses which in *A* 4–6 he had called, and 'proved' to be, inconcludent can in fact yield a conclusion, namely CoA, he expressly formulates only $AaB \& BeC$ and $AiB \& BeC$ of the *first* figure and utterly *ignores* the pairs $BoA \& BaC$ of the *second* and $AaB \& CoB$ of the *third* figures. These two pairs, and in general the pairs of the second and third figures which we named above, together with their conclusion CoA can *by virtue of this law* be transformed into moods of the same figure which Aristotle has already discussed and recognised as valid in *A* 4–6. Thus $BoA \& BaC \rightarrow CoA$ turns into $BaA \& BoC \rightarrow AoC$ (*Baroco*); $AaB \& CoB \rightarrow CoA$, applying the same principle, becomes $AoB \& CaB \rightarrow AoC$ (*Bocardo*); $BiA \& BeC \rightarrow CoA$ is changed to $BeA \& BiC \rightarrow AoC$ (*Festino*) etc. Only in the two *first* figure cases, $AaB \& BeC \rightarrow CoA$ and $AiB \& BeC \rightarrow CoA$ does the corresponding transformation fail to produce a recognised *first* figure mood; it gives instead two hitherto undiscussed moods: $BeA \& CaB \rightarrow AoC$ and $BeA \& CiB \rightarrow AoC$. Clearly this is the reason why Aristotle sets out explicitly only these two first figure moods.[16] They are, as it were, rehabilitated in our passage.[17]

However Aristotle's recognition of the validity of these moods produces an unpalatable dilemma: either he must recognise as concludent those premisses from which a proposition of the form CoA follows, in which case he must also admit that the method stated in *A* 4–6 for proving inconcludency does not suffice, since it leaves open the possibility that the 'inconcludent' premisses entail CoA – so that he must introduce a new method of proof; or else he can arbitrarily *stipulate* that implications with *o*-conclusions shall only be *called* syllogisms if they allow premisses of the prescribed form to yield a conclusion in AoC – in which case he

must recognise, in addition to the three figures discussed in *A* 4–6, a *fourth* figure to which the two last-named moods (*Fesapo* and *Fresison*) in fact belong. The question why Aristotle did not expressly recognise such a figure will be discussed in detail in § 25. The present considerations are intended only to show that Aristotle was aware of certain fundamental principles of the logic of relations and that it is thus not anachronistic to draw on relational logic in the explication of his definition of 'perfect' arguments.

Only in first figure syllogisms we find that the two relation-steps (the premisses) are immediately connected to each other. Although the same 'normed' ordering of the relational arguments can be set up in the other figures too, the cost of doing this is to render the relations of their syllogisms *heterogeneous*, since both non-converse and converse relations then appear. And we have shown that this fact would weaken the *evidence* of these syllogisms in exactly the same way as before.

§ 15. Evidence and Formulation

The preceding paragraphs discussed the formal features of first figure syllogisms on which their perfection, that is their evidence, depends; these in turn are dependent on the Aristotelian formulation of the premisses: "The *A* belongs to all *B*" etc., and are lost in the transition to the traditional formula: "All *B* is *A*". In the traditional *Barbara* "All *B* is *A*, all *C* is *B*: therefore all *C* is *A*" it is no longer the case that the first relation-step (from *B* to *A*) leaves off where the second (from *C* to *B*) begins. The middle term does not, as it does in the Aristotelian formulation, bind the premisses together in such a manner that the transitivity of the relation *a* is immediately transparent. Let us compare the formulation "All *C* is *B*, all *B* is *A*, all *C* is *A*", which is produced simply by *interchanging the two premisses*: we will agree at once that this is far more evident than the traditional formulation – indeed that it is just as evident as Aristotle's formulation: "If *A* is said of all *B* and *B* is said of all *C*, then *A* is said of all *C*". In fact, this formulation has the very formal properties which make Aristotle's *Barbara* a 'perfect' or 'evident' argument; the *traditional* formulation of *Barbara* does not have these properties and we must draw the almost paradoxical conclusion that Aristotle would not have attributed 'perfection' to the traditional *Barbara* – the evidence of which

logicians, in supposed obedience to Aristotle, have never tired of stressing. In fact, in traditional logic there is no difference of evidence between syllogisms of the different figures. Hence it became necessary either to base the privileged position assigned by Aristotle to the first figure on *other* grounds, or else to attack Aristotle's whole division of syllogisms into 'perfect' and 'imperfect' as purely arbitrary. Some logicians said simply that the arguments of the first figure even in the traditional idiom were 'evident' and that those of the others were not – without being able to define what they meant by the word "evident". Others spoke instead of the peculiar epistemological 'value' of the first figure. But logic is not interested in the epistemological 'value' of a proposition and in any case this has nothing to do with evidence. A third group of logicians, who had at least the merit of not seeing how the traditional syllogisms of the first figure were more 'evident' than those of the second, third, or fourth, attempted not to *justify* but at least to *explain* Aristotle's distinction by reference to his or Plato's metaphysics. Had it been seen that Aristotle's distinction between 'evident' and 'imperfect' inferences and his arrangement of the premisses within the figures is relative to his standard formulation, "The *A* belongs/does not belong to all/some *B*" all these manoeuvres would have been prevented.

It requires little effort to see this: on just those occasions where, instead of "be said of", "belong to", and "follow", Aristotle uses the expression "be in ... as in a whole" (*APr.* *A* 4, 25b33; cf. p. 9), and in place of "*A* belongs to all *B*" writes "*B* is in *A* as in a whole", where the order of the *terms* within the premisses is inverted, he also inverts the order of the *premisses*: "If the last is in the middle as in a whole and the middle in the first as in a whole, then there is a perfect syllogism with respect to the outer terms."[18] We have seen that outside his systematic exposition of syllogistic Aristotle sometimes presents syllogisms with concrete terms, in which the formulation by means of the copula "is" seems more natural. Chapter *A* 13 of the *Posterior Analytics* offers some good examples – and here Aristotle often transposes the premisses so that the transitivity of the relation remains completely clear, and hence the perfection of the argument is preserved. "If the planets do not twinkle, and what does not twinkle is near, then the planets must be near" (78a30–35). Contrast: "If anything that waxes in this way is spherical, and waxes the moon, then clearly the moon is spherical" (78b5–8). Here

the premisses are not transposed, and in addition the order of the terms in the second premiss is $P-S$. Aristotle did not always transpose the premisses when he employed a relational constant which, instead of the usual order of terms "$(A-B)\&(B-C)\rightarrow(A-C)$", licenses the order "$(B-A)\&(C-B)\rightarrow(A-C)$"; this is a pity but it can readily be explained: in such cases he was influenced by the order of premisses in the systematic enumeration of the syllogisms in *APr. A* 4–6. However, in *A* 4–6 there is *only one* passage where his choice of the constant "be in ... as in a whole" alters the order of the *terms*, and only here does he decide to alter the order of the *premisses* as well, – an order which he elsewhere invariably retains in first figure arguments. The corresponding change occurs sometimes in syllogisms with concrete terms: and this is proof enough that he was fully conscious that such an alteration in the order of the terms made the original evidence of the syllogism disappear, but that it could be brought back again by inverting the order of the premisses. In the same way Alexander transposes the premisses where he gives examples of syllogisms containing the copula or some other relational constant which has the same effect on the order of the terms: for example, "All men lie down, all that lies down sleeps; all men sleep" (*in APr.* 21. 21); "All men are capable of laughing, nothing which is capable of laughing is a horse; (no man is a horse)" (*in Top.* 2.10) – the latter is explicitly said to be a *perfect* syllogism of the *first* figure (ib. 2.7; 14). The later commentators, Philoponus and Ammonius, follow him conscientiously on this point. (Ammonius, *in APr.* 30, 35 sqq.; Philoponus, *in APr.* 36, 20; 78. 16 sq.; etc.)

§ 16. Order of the Premisses

The distinction drawn in § 14 between perfect and imperfect inferences was blotted out in traditional logic. However, the replacement of the constant "belong to" by the copula in the standard form of the premisses was not enough to obliterate it: transposition of the premisses would have preserved the evidence of first figure syllogisms even here. It will only disappear if the Aristotelian relation is supplanted by the copula and at the same time (by appeal to Aristotle) the normal order of the premisses as found in *APr. A* 4–6 is retained. The validity of an inference is obviously unaffected by the order of the premisses: we have already seen that the premisses of a syllogism, in Aristotle's view, make up the ante-

cedent of an implication, the consequent of which is the conclusion. The two premisses are joined by the logical constant "and"; they consist, that is, of a conjunction – and it is easy to see that the truth of a conjunction of propositions is not affected by the order in which the conjuncts are written. If the conjunction "All men are mortal and all Greeks are men" is true, then so too is the conjunction "All Greeks are men and all men are mortal" (cf. Whitehead and Russell, PM I, 116, prop. 4.3).

The controversy which raged in the 19th century as to whether the order of the premisses was relevant to the validity of the syllogism can thus easily be resolved. Waitz (I, p. 380), Prantl (I, p. 276) and Maier (SdA II., 1, p. 63), gravely maintained that the order of the premisses could not be changed without prejudice to the validity of the syllogism; Überweg (SdL^5, p. 330) and Trendelenburg (LU^3 II, p. 344)[19] held that the order of the premisses had no effect on the validity of the inference. Łukasiewicz (AS, § 12, pp. 32–34) rightly pointed out that "from the standpoint of logic the order of the premisses ... is arbitrary". However, our inquiry has shown that it is not by *chance* that Aristotle generally held to the order in which the first premiss contains the middle term and the predicate of the conclusion and the second premiss the middle term and the subject of the conclusion. For if the order of the premisses is of no account for the *validity* of a syllogism, it is extremely important to its *evidence*: the order of premisses chosen by Aristotle, supposing the formula "*A* belongs to *B*", leads in the first figure to evident syllogisms; this evidence disappears if the order of the premisses is altered. If the premisses are formulated by means of the copula, then they must be transposed if the first figure syllogisms are to be evident; with the traditional order of the premisses their evidence vanishes.[20]

In the case of the second and third figures, the order of the premisses does not of course affect their syllogisms' evidence either: nothing can raise them to evidence in Aristotle's sense. Aristotle's practice is in perfect accord with this fact. In his systematic treatment in *APr. A* 4–6 he maintains the standard order of the premisses for all valid syllogisms of the first figure, and again for all valid syllogisms of the second; in the third figure he alters the order of the premisses in the cases of *Felapton* (*A* 6, 28a26), *Disamis* (28b7), *Datisi* (28b11–12) and *Bocardo* (28b17–19). *Datisi* is in fact presented twice, once with the usual and once with the altered order (28b11 and 26). It can be concluded from this that Aristotle saw

that the order of the premisses was quite irrelevant to the validity of a syllogism, but recognised that the evidence of syllogisms of the first figure depended on that order. Thus the traditional order of the premisses is doubtless a 'convention' (Łukasiewicz, AS, p. 33). But it is a convention which Aristotle had good reason for establishing – and which traditional syllogistic, which formulates its propositions with the copula, ought to have *abandoned* for the very reasons which led Aristotle to *introduce* it, just as Aristotle himself abandons it when he employs an expression which, like the traditional "All *A* is *B*", alters the order of the terms within the premisses. Aristotle was followed in this (as will be shown in detail in § 19a and b) by the ancient commentators and authors of logical manuals – with the exception of Boëthius, the founder of traditional syllogistic.

§ 17. Perfect Modal Syllogisms

It has been pointed out (above, p. 44) that in Aristotle's view only first figure syllogisms but not all first figure syllogisms are 'perfect', i.e. evident. Membership of the first figure is thus for him a necessary but not a sufficient condition for the perfection of a syllogism. If we are to understand the precise meaning of the expression "perfect syllogism" in Aristotle's syllogistic, we must investigate those syllogisms which belong to the first figure but yet are not recognised as perfect inferences, and try to discover their common formal differences from the perfect syllogisms of the first figure. The formal properties which distinguish perfect syllogisms of the first figure both from syllogisms of the other figures *and* from imperfect syllogisms of the first figure must together make up the defining characteristics of a perfect inference. Only when we know these characteristics can we try to answer question (c) of p. 48: whether Aristotle's assertion that such syllogisms are substantially more evident than all other syllogisms is justified.

In assertoric syllogistic, that is, in the part of syllogistic in which premisses and conclusion are always propositions which contain no modal operator (possible, impossible, necessary; – Aristotle would say: no 'absolute' modal operator, cf. § 6, pp. 16–17), all syllogisms of the first figure are in Aristotle's view 'perfect'. In modal logic, however, while again only first figure syllogisms are 'perfect', not *all* are. Aristotle calls perfect those first figure syllogisms which are composed of three necessary propo-

sitions; and likewise those whose component propositions are all governed by the operator "possible". Further, syllogisms in which the *first* premiss and the *conclusion* have the same operator (whatever operator the second premiss may have) are perfect. Thus there remain those syllogisms in which the first premiss and the conclusion have *different* operators; and these Aristotle does *not* recognise as perfect. This fact has often been registered but never explained. I cannot here enter into the difficult problems of Aristotle's modal logic. Despite Becker's exemplary dissertation and the posthumous publication of Łukasiewicz' investigations (in AS², Oxford, 1957), it has not yet been fully illumined.[21] However, to solve our present problem, the meaning of the expression "perfect inference", we must try to understand *why* Aristotle denied perfection to these particular first figure modal syllogisms. We limit ourselves to syllogisms in *Barbara*, since what we say about them will always hold *mutatis mutandis* for the other syllogisms of the first figure.

Becker maintained that Aristotle did not conceive the operator on a proposition of modal logic as governing the whole proposition but as determining the *predicate* of the proposition. This interpretation is not wholly immune to objection, but we shall nevertheless use it as the foundation of our present discussion. As a result of this decision, we shall use the letters *N*, and *P*[22] for the modal operators "necessary", and "possible" and prefix them to the predicate-variables of the propositions which Aristotle treats as 'necessary' and 'possible' (in the traditional vocabulary "apodeictic", and "problematic" propositions). Thus, remembering our earlier remarks, we may produce as a perfect syllogism in the form *Barbara*:

(1) $NAaB \,\&\, NBaC \rightarrow NAaC$ (*A* 8, 29b36–30a5)

or

(2) $NAaB \,\&\, BaC \rightarrow NAaC$ (*A* 9, 30a17–23).

In fact only the second of these syllogisms has the property which we have discussed before and have seen to generate evidence – that is, *identity* of the last member of the first relation and the first member of the second. But we must grant Aristotle that *NBaC evidently entails BaC*, so that the transition in the first syllogism can be made just as clear. The same holds for syllogism

(3) $AaB \,\&\, NBaC \rightarrow AaC$ (*A* 9, 30a23–32).

Similarly

(4) $PAaB \& PBaC \rightarrow PAaC$ (*A* 14, 32b38–33a1)

and

(5) $PAaB \& BaC \rightarrow PAaC$ (*A* 15, 33b33–36)

are called perfect. Syllogism (5), in particular Aristotle's assertion that it is evident, poses no problem. (4), however, presents us with certain difficulties: here again the last member of the first relation and the first of the second do *not* in the strong sense coincide, for *PB* and *B* are not identical predicates; moreover there is an additional difficulty: *PBaC* does not evidently entail *BaC* – it evidently does *not* entail it. This syllogism's claim to evidence must therefore, according to Aristotle's own stipulations, be *rejected*. It is precisely in this passage that Aristotle introduces a new definition of "ἐνδέχεσθαι ὑπάρχειν" and of the statements in which this operator occurs: this definition is specially adapted to *produce* the missing identity of the last member of the first and the first of the second relation. In his new analysis of possibility assertions Aristotle says that the proposition "*A* can belong to all *B*" may be taken as equivalent to the proposition "*A* can belong to all things to which *B* belongs" (*PAaB*), but that it may also be construed as "*A* can belong to all things to which *B* can belong" (*PAaPB*) (*APr. A* 13, 32b23–37).

He explicitly adopts for his subsequent investigations the meaning *PAaPB*; but only for those cases in which it is necessary to produce a smooth transition from the first to the second premiss. Syllogism (4) then has (assuming this interpretation or definition) the form: $PAaPB \& PBaC \rightarrow PAaC$, and is therefore, on this assumption, a perfect syllogism in Aristotle's sense. Thus Aristotle says that (4) is φανερὸν ἐκ τοῦ ὁρισμοῦ· τὸ γὰρ ἐνδέχεσθαι παντὶ ὑπάρχειν οὕτως ἐλέγομεν (32b40–33a1). Aristotle does not say here – and Becker (ATM, pp. 33–37) is quite right to reproach him for this – which of the two interpretations he wants to underlie his modal logic. He clearly falls back on the second interpretation only in order to get round the difficulty we have noticed. All this confirms our belief that the perfection of an argument essentially depends, in Aristotle's view, on the 'identity' of the final member of the relation which forms the content of the first premiss with the opening member of the relation which the second premiss asserts to hold.

In opposition to these modal syllogisms in *Barbara* all of which Aris-

totle calls 'perfect', he explicitly calls *imperfect* syllogisms of the form:

(6) $AaB \& PBaC \rightarrow P1AaC$[23] ($A$ 15, 34a34–b2)

and

(7) $NAaB \& PBaC \rightarrow P1AaC$ (A 16, 35b38–36a2).

On the other hand the last remaining syllogism in *Barbara*

(8) $PAaB \& NBaC \rightarrow PAaC$ (A 16, 36a2–5)

he again allows to be perfect (36a5) – clearly because here too "$NBaC$" evidently entails "BaC" (cf. p. 62). The question why Aristotle calls syllogisms (6) and (7) imperfect in spite of the fact that they belong to the first figure, has been answered by those few commentators who have bothered to pose it (e.g. Alexander, 174, 20–30, Ross, APPA, p. 336) in basically the same way. Alexander explains the imperfection of these syllogisms by the following consideration: in a *perfect* syllogism what holds of B holds of C because C is part of B. The first premiss establishes what holds of B and the second that C is in fact B. In the case of syllogisms (6) and (7) it is not true that C *is* B but only that it *can* be (and therefore can also not be) B; and for this reason the necessity of the progress from A via B to C is not at once evident. Ross uses the same argument in a more formal manner when he points out that "'All B is A, all C is capable of being B' are premisses that in their present form have no middle term" (APPA, p. 336). He here appoints "capable of being B" as middle term of the second premiss, and "B" as middle term of first; and this accords with Aristotle's practice, which Ross states on page 319, of treating the modal operator of a proposition as part of the predicate. However, both Alexander and Ross have overlooked the following point: syllogism (4), $PAaB \& PBaC \rightarrow PAaC$, has the very formal property which they adduce to explain why Aristotle held (6) and (7) to be imperfect – and (4) is expressly recognised by Aristotle as a 'perfect' inference (32b39).

In what other way does (4) differ from (6) and (7)? Or rather: in what way do (6) and (7) differ from all perfect syllogisms? Clearly only in that the term $P1A$, predicate of C in the conclusion, is here *not* identical with the major term of the first premiss (A or NA). In (5) for example PA appears in both places: in (6) on the other hand we have A and $P1A$, and in (7) NA and $P1A$. Thus it is not the non-identity of the *middle term*, but the non-identity between the predicate of the first premiss and the predi-

cate of the conclusion which is in Aristotle's view a sufficient reason for calling the syllogisms in question non-evident and hence imperfect. And it is clearly with reason that Aristotle will not recognise as evident any syllogisms which deviate from the schema according to which one predicate *A*, which belongs to *B*, also belongs to *C* if *B* belongs to *C*. On the other hand it is plain that the cut between 'perfect' and 'imperfect' syllogisms as made in these passages is somewhat *arbitrary* in its results. Every unprejudiced mind will find syllogisms (2) and (5) substantially more evident than (1), (3), (4) and (8). Becker too (ATM pp. 32–33, 41) found the evidence of (1) and (4) highly dubious. We would be inclined to connect (1), (3), (4) and (8), as far as evidence goes, with (6) and (7) rather than with (2) and (5). For, within the framework of Aristotle's terminology (as we showed in § 12) while it does not make sense to speak of degrees of evidence, it does to speak of degrees of *non*-evidence. Nevertheless, we can explain why Aristotle did not draw the boundary between perfect and imperfect syllogisms in such a way as to divide (2) and (5) from (1), (3), (4), (7), and (8): he must have thought it absurd to suppose that the *identity* of the operators on all the propositions of a syllogism could *impair* its evidence, so that a syllogism with three necessary components like (1) could be less evident than the same syllogism with an assertoric minor, (2), and a syllogism with three problematic propositions as premisses and conclusion like (4) could be less evident than one with a problematic major and assertoric minor like (5). However, it is a mere prejudice, no doubt a seductive one, to regard modal conformity of premisses and conclusion as the source of special evidence.

The alternative classification of the modal syllogisms in *Barbara* which we have suggested and which differs from Aristotle's in that it classes all arguments except (2) and (5) as 'imperfect', has the advantage of a stronger analogy with the division of *assertoric* syllogisms into 'perfect' and 'imperfect' arguments. Just as Aristotle defines 'imperfect' assertoric syllogisms as those needing "one or more" operations to make them evident (*A* 1, 24b25), operations which (as we shall later argue in detail: §§ 28–29) are based on certain laws of *predicate-logic*; so we might here define the 'imperfect' syllogisms of the *first* figure as those which only become evident after "one or more" operations based on rules of *modal logic* – more exactly, operations based on certain implications which hold between the modal operators. (1), (3) and (8) become evident if their

second premiss *NBaC* is changed, by virtue of the implication (I) *NAaB* →*AaB*[24], into *BaC*; (4) becomes evident if its first premiss *PAaB* is put in the form *PAaPB*. Aristotle expressly (32b25–30) allows this, and so we may use a further implication of his modal logic, namely (II) *PAaB*→ *PAaPB*.

(6) and (7) remain. In the conclusion of both the operator "possible" differs from the normal one. Elsewhere in Aristotle the possibility operator is two-sided: i.e. a proposition is possible if it is (a) not necessary and (b) not impossible. But, because the first premiss of (6), "*A* belongs to all *B*", does not *exclude* the possibility that *NAaB* also holds, while in (7) *NAaB is* the first premiss, and in both syllogisms the second premiss states that all *C* is possibly *B*, in neither case is it excluded that all *C* is *necessarily A*: hence the conclusion here cannot take the usual operator, but only the weaker[25] one "not impossible", which Becker symbolises by "E_1" and which I here express as "*P*1". To turn (6) and (7) into perfect syllogisms we therefore need the implication (III) *AaB*→*P*1*AaB*; and this in fact holds – what is actual is not impossible.[26] The implication *AaB*→ *PAaB*, on the contrary, does *not* hold; for what is actual is not thereby "neither impossible *nor* necessary": it may be necessary.[27] By *AaB*→ *P*1*AaB*, (III), I transform (6) into (6′): *P*1*AaB*&*PBaC*→*P*1*AaC*; and by (II) (6′) becomes (6″): *P*1*AaPB*&*PBaC*→*P*1*AaC*. (6″) now satisfies all Aristotle's criteria for perfect modal syllogisms. In the case of (7) I need (I), in addition to (II) and (III), in order to transform *NAaB* into *AaB*; otherwise the procedure is the same as with (6).

Thus (1), (3), (4), and (8) each need one modal operation to become perfect syllogisms ((1), (3), and (8) an operation based on law (I) *NAaB*→ *AaB*; (4) one based on law (II), *PAaB*→*PAaPB*); (6) needs two operations based on laws (II) and (III); (7) three, based on (I)–(III). Syllogisms (6) and (7), the only ones which Aristotle calls 'imperfect', do as a matter of fact differ from the others as regards their evidence: each needs more than one operation before it is transformed into a perfect argument. This may be taken as evidence that Aristotle did in fact pursue the train of thought we have sketched. There is only one difference, probably due to the prejudice we have mentioned above: in his modal logic Aristotle allows perfection to syllogisms whose evidence is established by one of the operations we have described, whereas the alternative division which we have proposed here is in exact formal agreement with

the one Aristotle chooses for assertoric syllogisms. Moreover, our division alone fits Aristotle's terminology, in which all evident syllogisms must be equally evident, while imperfect syllogisms can always be more or less imperfect (cf. pp. 48–49). Aristotle's division of the modal syllogisms of the first figure includes among the evident syllogisms some which are *more* evident – (2) and (5) – and some which are *less* evident – (1), (3), (4), and (8) –; further, among the nonevident, and therefore imperfect, arguments, one – (6) – is less and one – (7) – is more imperfect. In assertoric logic too there are more and less imperfect syllogisms, according to whether they need one or more (ἑνὸς ἢ πλειόνων) operations to establish their evidence. For example, *Festino* (II) can be changed into *Ferio* (I) and thus made evident in one operation; *Camestres* (II) and *Disamis* (III), on the other hand, need two operations (or three, if the so-called 'Metathesis Praemissarum' is to be counted as a separate operation as it is in traditional logic) before they are transformed into *Celarent* (I) and *Darii* (I), and thus into evident syllogisms. We shall see that Theophrastus drew certain conclusions from this fact (cf. pp. 74–75). The syllogisms of the first figure are, however, in Aristotle's view, 'evident' in just the same manner and to just the same degree – at least in the framework of *APr. A* 4–6.

These results enable us to answer question (c) of page 48: our investigations have shown that when Aristotle designates certain syllogisms as 'perfect', he is asserting that they are substantially more transparent than all other syllogisms: this assertion is false. It is false that syllogism (4), say, is substantially more transparent than, say, assertoric *Festino*. In both cases one operation based on a logical law is required in order to establish evidence; it makes no difference that the law in the one case belongs to *predicate*, in the other to *modal* logic. The assertion would be justified if, in accordance with our proposal, Aristotle were to allow only (2) and (5) to be 'perfect' arguments: these alone have the formal properties which, in his assertoric logic, Aristotle treated as necessary and sufficient conditions for the 'perfection' of a syllogism.

§ 18. *Prior Analytics A* 7 and Aristotle's Modal Logic

At this point I should like to offer a conjecture, which, if it is correct, would shed light on the genesis of Aristotle's logical system. As we have

shown, in chapters *A* 4-6 of the *Prior Analytics* Aristotle does not distinguish between the four moods of the first figure (*Barbara*, *Celarent*, *Darii* and *Ferio*) with regard to their evidence. However, in the second section of chapter 7 he presents a detailed argument for the correct thesis that *Darii* and *Ferio* can be reduced to *Celarent*, and hence that all syllogisms can be reduced to *Barbara* and *Celarent* (29b1–25). Bocheński[28] believes this thesis to be a discovery made after the writing of *A* 4–6; and Łukasiewicz (AS, p. 45) supposes the reason for this innovation to be that Aristotle (although he regarded *all* first figure syllogisms as 'perfect') wanted to use as axioms of his system only the "most clearly evident" (the 'evidently evident'?) syllogisms of the first figure, namely *Barbara* and *Celarent*. Łukasiewicz here speaks of a difference in evidence between on the one side *Barbara* and *Celarent* and on the other *Darii* and *Ferio*. Philoponus (*in APr.* 114, 16–22, ad 29b1) makes the same point: in this passage Aristotle δείκνυσι τοὺς δύο τοὺς πρώτους (sc. συλλογισμοὺς) καὶ τῶν λοιπῶν τῶν ἐν τῷ αὐτῷ (sc. τῷ πρώτῳ) σχήματι τελειοτέρους: "he shows that the first two syllogisms of the first figure are more perfect than the two other syllogisms of the same figure". We could understand without any trouble the claim that *Barbara* – alone – is more perfect than the other first figure syllogisms: as we noted above, only *Barbara* consists entirely of *a*-propositions, and *a* is the only transitive relation between syllogistic terms. But if both *Barbara and Celarent* are to be more perfect, then there must be a (different) formal property common to both syllogisms, that raises them above *Darii* and *Ferio*. A solution to this problem seems to offer itself if we consider what Aristotle does in *modal* logic, where even among syllogisms of the *first* figure there exist different degrees of evidence: all the first figure syllogisms which Aristotle there calls perfect are formally distinguished by the fact that the modality of their first premiss and that of their conclusion is the same. In some of these syllogisms the second premiss too has the same modality – (1) and (4) – in others it is deviant – (2), (3), (5), and (8). By contrast, in the two syllogisms which Aristotle here calls 'imperfect' the conclusion has a different modality from that of the first premiss. This suggested that 'perfection' should be ascribed only to syllogisms in which the relation between *A* and *B* can, by the mediation of that between *B* and *C*, be 'transmitted' or 'conveyed' to the pair *A* and *C*. And this stronger requirement is only satisfied by *Barbara* and *Celarent*: *Darii* has *a* in the

first premiss and *i* in the conclusion, and *Ferio* has *e* in the first premiss and *o* in the conclusion. That is to say, Aristotle had laid down in his modal logic (which all scholars place after the assertoric, and which indeed presumes it expressis verbis) that not *all* syllogisms of the first figure have those formal properties which bestow Aristotelian 'evidence' on an argument. This suggested the drawing of an analogous distinction between two groups of first figure syllogisms inside *assertoric* logic; and *A* 7, which is manifestly a later addition to *A* 4–6, provided the proof, which *then* became quite indispensable, that all syllogisms can be reduced to evident arguments even if the conditions for evidence are given this added stringency.

§ 19. Historical Excursus

a) *Antiquity*

It is in many ways instructive for the student of the history of logic to consider the fate of Aristotle's distinction between 'perfect' and 'imperfect' syllogisms. His school, even in the first generation after his death, engaged in lively discussion on the topic. 650 years later it was a bone of contention among the scholars at the court of the Emperor Julian. In the Middle Ages the doctrine was accepted on the mere authority of Aristotle's name, even though the traditional form of the syllogism had made it quite impossible to comprehend. Later it became the slogan of a clumsy polemic against the validity of syllogisms of the second, third and fourth figures. Finally, in the 19th century it was exalted to the position of the clamp which bound together logic and metaphysics, only to be reduced again to a historical curiosity and explained in terms of the 'history of ideas'. Texts which bear witness to this Odyssey will now be presented. Commentary can be brief, since all that is necessary to judge the opinions propounded has been set out in the preceding pages.

According to the testimony of *Ammonius* – c. 490 AD – in his commentary on the *Prior Analytics* (31, 11 sqq.), the Neoplatonic philosophers Porphyrius (d. c. 300 AD), Iamblichus (d. c. 330) and Maximus Thaumatourgos (d. 370) a pupil of Iamblichus' disciple Hierios, disagreed with Aristotle and recognised the second and third figure syllogisms as perfect too. Ammonius tells us that these thinkers, in particular Porphyrius, founded their opinion on the authority of Boëthus (the head of the

Athenian Peripatos, c. 40 BC) and perhaps also on that of Theophrastus. Ammonius' teacher Proclus (410–485) and Hermias, his father and Proclus' colleague, followed the majority, and their view seems to have represented Neoplatonic orthodoxy. It was opposed by a group of logicians (whom Ammonius refers to as τινές, 30.32 sqq.; 31.26) who held that Aristotle did not regard the imperfect syllogisms as valid arguments. Ammonius had little trouble in finding cogent counter-arguments to this interpretation. Since he does not number Themistius (320–390) among this group, we might see in Themistius an advocate of the genuine Aristotelian doctrine. For we hear later from Ammonius that Themistius attacked the Neoplatonic interpretation, in particular that of Maximus. The controversy between the two scholars was finally decided in favour of Maximus by Julian's imperial arbitration – as might have been expected both from the high favour Maximus enjoyed at court and from Julian's ignorance of logic.

Ammonius, Philoponus' teacher, shows his own acuity by giving a list of erroneous assumptions which had been drawn on in the long and protracted controversy. He points out, for example, first, that in his distinction between 'perfect' and 'imperfect' syllogisms Aristotle claims definer's freedom (καὶ πάλιν οὐ λέγει "τέλειος δέ ἐστιν" ἀλλὰ "τέλειον δὲ καλῶ καὶ ἀτελῆ" ὡς αὐτὸς ὢν ⟦φησιν⟧ τούτων ὀνοματόθετης: 32.26–28); and secondly, that Aristotle says imperfect syllogisms need something else, not to become necessary, but to become evidently necessary (32.30–33). Of course, Ammonius' own position deviates from Aristotle's: he holds that *all* syllogisms are perfect (ἀλλ' οἴεται μὲν αὐτὸς εἶναί τινας ἀτελεῖς τῇ δὲ ἀληθείᾳ πάντες τέλειοί εἰσιν: (14.32 sq.). His argument is that all so-called 'imperfect' syllogisms already have all the ὅροι that a syllogism, and a perfect syllogism, stands in need of; that their ὅροι are only jumbled together (μόνον συγκεχυμένοι εἰσὶν ··· οἱ ὅροι: 33.21). However, this argument can only be taken as an objection to Aristotle if we suppose that ὅρων must be understood as the supplement to προσδεόμενων ἢ ἑνὸς ἢ πλειόνων (24b24). For only then is Aristotle's definition incorrect. However, if we pay proper attention to Aristotle's careful language in this passage, it is clear that he did not mean to say that imperfect syllogisms lack a ὅρος. He is thinking rather of the logical *operations* (e.g. conversion) by the help of which every valid syllogism can be transformed into a perfect argument.

If Ammonius' notice (31, 22–5) is correct, and Theophrastus had already, in opposition to Aristotle, recognized the second and third figure syllogisms as perfect, then this may be related to the fact that, while Aristotle only treated the additional moods of the first figure (cf. § 14) in an appendix to his systematic exposition of syllogistic, Theophrastus explicitly fitted them into the system as the fifth to ninth moods of the first figure. We know this from Alexander's commentary (*in APr.* 69, 26 sqq). Alexander adds – and his text compels us to believe that he is quoting Theophrastus – that these new additional moods of the first figure are not 'perfect'.[29] If the first figure contains imperfect syllogisms, then the two other figures could be regarded as on a level with it. At any rate, Theophrastus, and thus the first generation of Aristotle's successors, did not understand Aristotle's real intentions on this question. By contrast, it can be proved that later commentators possessed a correct understanding of Aristotle's doctrine. The passages in Alexander (c. 200 AD) and Philoponus (6th century AD), in which Aristotle's position is, if not explained, at least clearly exposed, will shortly be adduced. Almost more important than this direct testimony is the fact we have already touched on that both Alexander and Philoponus very often formulate first figure syllogisms by means of the *copula* instead of Aristotle's "belong", and in these cases always change the order of the premisses. This practice can only be explained, as far as I can see, by supposing that the authors realised that it alone could preserve the evidence of the syllogisms. Whoever realises this can only understand Aristotle's term "evidence" in the sense we have worked out in §§ 11–18. This practice was so influential that it bound even those authors who themselves no longer recognised the distinction between perfect and imperfect syllogisms. This is shown by the example of Ammonius, whose pupil Philoponus far excelled him in logical ability.

Alexander of Aphrodisias, in his discussion of Aristotle's definitions of 'perfect' and 'imperfect' inferences, states emphatically that imperfect syllogisms are none the less *valid* syllogisms. Surprisingly, however, he does not, like Ammonius[30], cite as evidence for this the difference between γενέσθαι and φανῆναι τὸ ἀναγκαῖον (24b22–24), but says, obscurely enough, that in 'imperfect' syllogisms, as in enthymemes – in which a premiss is suppressed – there is something missing; however, this missing element – in contrast to the case of enthymemes and other non-syllogistic

arguments – is contained as it were 'potentially' in the premisses, so that imperfect syllogisms need to be, not supplemented, but "supported" (βοηθεία) and "unveiled". The conclusions of perfect syllogisms are evident provided only that we know the definitions of "be said of all" and "be said of none" (174.6 sqq.). Here Alexander is relying on an explanation of Aristotle's (*APr. A* 4, 26a24), according to which a syllogism is evident if nothing more than knowledge of the definitions of the constants *a* and *e* is required to understand it (*i* and *o* are used in the definitions of *a* and *e*: *A* 1, 24b28–30). However, that is only to say that anyone who understands the propositions in which these constants occur, will also recognise the whole syllogism as valid: the statement offers no glimpse of how such a syllogism as it were manages to be immediately transparent. And without such information Alexander's assertion that *Barbara* and *Celarent* are 'more perfect' than *Darii* and *Ferio* and that *therefore* Aristotle shows in *APr. A* 7 how all the moods can be reduced to the two first moods of the first figure, is lacking a reliable basis (113.7–15). We cannot say that Alexander took much trouble to reach a full understanding of Aristotle's definition of perfect syllogisms. What he says is only a rearranged quotation of Aristotle's own text, more paraphrase than commentary.

Philoponus' discussion (*in APr.* 36, 19–37.2) reflects a more enterprising and more independent mind. It seems to represent the highest point reached by the ancient commentators on our question. For this reason I quote him at length:

36, 19 τέλειος μὲν οὖν συλλογισμὸς οὗ τὸ συμπέρασμα παντός ἐστιν ἐπενεγκεῖν, ὡς ἂν εἴπῃ τις "τὸ δίκαιον καλόν, τὸ καλὸν ἀγαθόν". ἐνταῦθα γὰρ παντός ἐστιν ἐπακοῦσαι "τὸ ἄρα δίκαιον ἀγαθόν". ἀτελὴς δὲ οὗ τὸ συμπέρασμα μόνου ἐστὶ τοῦ ἐπιστήμονος ἐπαγαγεῖν, οἷον "πᾶς ἄνθρωπος οὐσία, πᾶς ἄνθρωπος ζῷον". τούτου τὸ συμπέρασμα τὸ τὶς ἄρα οὐσία ζῷον". καὶ ὅτι τοῦτο, ἔστι ῥᾴδιον φανῆναι διὰ τῆς ἀντιστροφῆς. εἰ γὰρ ἀντιστρέψωμεν καὶ οὕτως εἴπωμεν "τὶς οὐσία ἄνθρωπός ἐστι, πᾶς ἄνθρωπος ζῷόν ἐστι" δῆλον ὅτι συνάγεται "τὶς ἄρα οὐσία ζῷόν ἐστι". διὸ καλῶς οὐκ εἶπε "πρὸς τὸ γενέσθαι" ἀλλὰ "πρὸς τὸ φανῆναι". οἱ μὲν γὰρ τέλειοι συλλογισμοὶ καὶ τὸ ἀναγκαῖον ἔχουσι καὶ τοῦτο φαινόμενον· οἱ δὲ ἀτελεῖς, ὁποῖοί εἰσι πάντες οἱ ἐν τῷ δευτέρῳ καὶ τρίτῳ σχήματι, ἔχουσι μὲν τὸ ἀναγκαῖον, οὐ μὴν προφανές, ἀλλ' ἐπι- 37, 1 στήμονος δέονται, ἵνα τὸ ἐκ τῶν προτάσεων εἰλημμένον ἀναγκαῖον ‹μὲν› μὴ φαινόμενον δὲ εἰς τὸ ἐμφανὲς ἀγαγῇ διὰ τῶν ἀντιστροφῶν.

"A perfect syllogism is one the conclusion of which everybody is able

to draw; as if someone said "the just is beautiful; the beautiful is good" – for here anyone can understand (ἐπακοῦσαι) "therefore the just is good".[31] An imperfect syllogism is one the conclusion of which a logician (ἐπιστήμων) can draw, e.g. "every man is substance; every man is animal". The conclusion of this is: "Therefore some substance is animal". That this is the conclusion can easily be made apparent by the use of conversion. For if we convert and say "some substance is man; every man is animal" then it is obvious that "some substance is animal" follows. For this reason he (sc. Aristotle) rightly said, not γενέσθαι τὸ ἀναγκαῖον, but φανῆναι τὸ ἀναγκαῖον. For perfect syllogisms both have necessity and evidently have it; imperfect syllogisms, such as all those of the second and third figures, have necessity but do not have it evidently: they need a logician to take the necessity which comes from the premisses but is not evident, and lead it into the light by means of conversions." In this passage the difference between perfect and imperfect inferences is well presented as a difference of *evidence*; it is a nice idea to make the distinction between degrees of evidence sharp and immediate by referring to two classes of thinkers, logicians and the class of all men: an argument is evident if it is transparent to anyone; non-evident in Aristotle's sense if it is clear only to a logician.

However, in the other passages where Philoponus comments on Aristotle's use of the expressions "τέλειος" and "ἀτελής" (Index CIAG XIII, 3, 527 sq. and 459) we find two ideas which contain or at least foreshadow some of the misunderstandings of the later tradition. The first of these notions is that imperfection should be so defined that a syllogism is imperfect if and only if Aristotle offers a *proof* of it in the *Analytics*: e.g. 195.21–22: φανερὸν δὲ καὶ ὅτι ἀτελής ἐστιν οὗτος ὁ συλλογισμός, εἴγε διὰ τοῦ ἀδυνάτου δέδεικται. But Aristotle does not call a syllogism imperfect because he gives a proof of it: he gives a proof of it because it is imperfect. Indeed, in *A* 7 he gives proofs for perfect syllogisms too (*Darii* and *Ferio*) and still expressly calls them perfect. This misunderstanding led Maier (SdA II, 2, p. 120) to maintain that Aristotle did not recognise syllogisms of the form "$NAaB \& BaC \rightarrow NAaC$" (our (2), § 17) as perfect, although the addition of φανερὸν (*APr. A* 9, 30a22) explicitly designates them as such. Maier took the passage, which states briefly why such a syllogism is *perfect*, as a proof of the *validity* of the syllogism; and then, arguing from the false maxim that a syllogism which Aristotle

proves is eo ipso not perfect, inferred that (2) must in Aristotle's view be an imperfect syllogism. And in the same passage Maier reproaches Aristotle for not recognising the argument as perfect.

The second idea – a natural one which had far-reaching consequences for the future of logic – appears twice in Philoponus (72.23 and 298.27): the predicate τέλειος is transferred from its original subject, "syllogism", to the "figure" to which perfect syllogisms belong. On page 72 Philoponus remarks that *Darii* and *Ferio* do not properly fit the requirements for evidence which Aristotle formulates at *APr. A* 4, 25b32–35. For in these two arguments it is not in fact true that "*C* is in *B* as in a whole and *B* is or is not in *A* as in a whole", and yet Aristotle says that syllogisms whose terms are so interrelated are perfect. To explain how Aristotle can nevertheless explicitly call the two moods perfect (26a20; 28), either we must suppose that in 25b32–35 he does not mean that *only* such syllogisms are evident, but is for the moment interested only in *Barbara* and *Celarent* which he formulates in the immediately following sentence; or else we must find some reason why he called *Darii* and *Ferio* perfect, even though they do not answer to his 'definition'. It is clear from Aristotle's text that only the first supposition is possible in the context. Philoponus, however, makes the second supposition, and offers as the 'reason' for Aristotle's procedure (which he is then of course bound to discover), the notion that he calls the two moods perfect because "they preserve the perfect form of the first figure" (τοὺς σώζοντας τὸ τέλειον εἶδος τοῦ πρώτου σχήματος: 72, 30). We have already shown (p. 44) how misleading it is to speak of a perfect *figure*. If Philoponus uses this mode of expression (e.g. later, 298, 33 sq.: ταύτῃ γὰρ καὶ ἀτελῆ λέγονται εἶναι τὰ σχήματα, τὸ δεύτερον καὶ τὸ τρίτον, διότι μὴ προφανῆ τὴν ἀνάγκην ἔχουσιν), we can at least say in his favour that he thought this terminology and the notion lying behind it could solve a difficulty in Aristotle's text – albeit a difficulty which, we have seen, is of his own making. Later authors can claim no such motive – which is at least rational in itself – for their adoption of the same idiom. It is interesting that Alexander clearly remained truer to Aristotle's position. In his works the expressions "τέλειος" and "ἀτελής" (cf. Index CIAG II, 2, 288 and 611) do not once appear as predicates of "figure"; nor does he ever say that a syllogism is imperfect because it is proved; and, finally, he never allows himself, as Philoponus does, to call the first two moods of the first figure

"more perfect" than the last two, although he states emphatically (77, 6–9 and 26–28) that it is reasonable (εἰκότως: 27) to call syllogisms which need *two* conversions before they are transformed into first figure arguments, "*more im*perfect" than those which need only one conversion. This is fully consonant with Aristotle's terminology, and probably derives from Theophrastus, who, we are told (Prantl I, p. 369, n. 48), inverted Aristotle's order of *Disamis* and *Datisi* in the third figure, because *Datisi* needs only one conversion whereas *Disamis* needs two (cf. Philoponus *in APr.* 105, 28–31).

All the Greek commentators show that they understand, as it were de facto, Aristotle's distinction between perfect and imperfect syllogisms, in that they regularly alter the order of the premisses in the first figure (and often in the other figures too: references follow shortly) when, instead of "*A* belongs to all *B*" they use "All *B* is *A*" or a formally equivalent expression. I can only explain this practice by supposing that they understood Aristotle's notion of evidence and desired to preserve the evidence of the first figure syllogisms even when they were formulated in the later idiom. When this practice died out, the nature of Aristotle's distinction was no longer understood. Since our sources for the history of logic in late antiquity are sparse, we must be cautious in assigning to *Boëthius* the dubious honour of founding the scholastic tradition: this, by adopting the 'Aristotelian' order of the premisses and using the 'non-aristotelian' copula in place of the relation "belong to", blocked off all access to the distinction in question; nevertheless, it took over and maintained, on Aristotle's authority, the assertion that first figure syllogisms are perfect and therefore worked out new and completely different reasons for the preeminence of this figure. It is well-known that Boëthius' logical treatise was, in the Latin West, the main source of logical studies until the Middle Ages. In it we find for the first time the traditional form of *Barbara*: "Omne iustum bonum est. Omnis virtus iusta est. Omnis igitur virtus bona est." (*Patrol. Lat.* ed. Migne, vol. 64 (1891), p. 822). At the end of his enumeration of all twenty moods of the three figures, Boëthius says: "Hi sunt igitur omnes trium figurarum modi quorum primae figurae quatuor primae (clearly a misprint for "primi") indemonstrabiles nominantur et directi, id est sine aliqua conversione monstrati; indemonstrabiles autem, quoniam non per alios demonstrantur, et perfecti dicuntur, quoniam per seipsos comprobantur. Et primi quoniam positione et natura

primi sunt, et in eos omnes caeteri resolvuntur" (823A). The affirmation that the perfect syllogisms "per seipsos comprobantur" rings harsh and dogmatic, and the very language ("arguments which prove themselves"?) sounds unaristotelian. It is thus hardly surprising that the *Summulae Logicales* of Petrus Hispanus, the influential text-book of the 13th century, passes over the doctrine of perfect syllogisms in silence.

Boëthius' Latin precursors, however, Apuleius Madaurensis (a contemporary of Galen) and Martianus Capella (c. 470 AD), adopted the practice of the Greek commentators from the logical compendia they used and so left Aristotle's distinction between evident and non-evident, and hence 'perfect' and 'imperfect', syllogisms with its own proper meaning. Apuleius set out the premisses in the inverted order – and that not only in the first but in all the figures. (This does not matter, since the order of the premisses is important only in the first figure; indeed it is an improvement, since the second and third figure syllogisms can be transformed into the corresponding moods of the first figure without further ceremony, simply by performing the conversions which Aristotle prescribed.) *Barbara* runs, in his text (*de Interpretatione* IX; Apuleius, *Opera* IV, ed. Thomas, Teubner (1908), p. 186; quoted in Prantl I, p. 587, n. 19): "Omne iustum honestum, omne honestum bonum, omne igitur iustum (istum, Prantl) bonum est." On the assumption of this mode of formulation Apuleius says, correctly and clearly enough: "Ex hisce igitur in prima formula (=figura) modis novem (the Theophrastian moods of the first figure; cf. above p. 70) primi quatuor indemonstrabiles nominantur (nominentur, Prantl I, p. 588), non quod demonstrari nequeant, ... sed quod tam simplices tamque manifesti sunt, ut demonstratione non egeant" (l.c. 188).

Prantl thinks it a "strange idiosyncrasy" of Apuleius' that he "consistently sets out the premisses in a perverse order" (I, p. 587). If this "perverse" order did not also occur in Pseudo-Galen (cf. Prantl I, p. 599, n. 39) and in Martianus Capella (Prantl I, p. 678) and Cassiodorus (I, p. 723), Prantl would be inclined to suppose a simple mistranslation on the part of Apuleius (which moreover cannot explain what Prantl thinks it ought to explain). It has quite escaped Prantl that the very order of the premisses which he censures is already found in the Greek commentators – indeed in Alexander whom he so rightly extols: "Every man is capable of laughing; nothing that is capable of laughing is a horse; (no man is

a horse)" – first figure –; "Every man is capable of laughing; no horse is capable of laughing; (no man is a horse)" – second figure –; "All that is capable of laughing is a man; nothing that is capable of laughing is a horse; (no man is a horse)" – third figure (*in Top.* 2, 9sqq.). All these concrete syllogisms exhibit the 'perverse' order of premisses. Philoponus gives examples for each of the three figures at 67.30–68.10; first figure: "Man is an animal; animal is a substance; therefore man is a substance". Philoponus himself (67, 34 sq.) calls "animal is a substance" the *major*, and "man is an animal" the *minor* premiss of the syllogism. In his view, therefore, the major remains the major even when the premisses are transposed. Second figure: "Animal is said of every man, animal is said of no stone; therefore man is said of no stone". Here the usual order of the premisses of *Camestres* is preserved – and so is the usual relation "be said of". Third figure: "Some stones are white; every stone is inanimate; therefore some white things are inanimate." The *correlation* between the order of the premisses and the expression used to symbolise the relation between the terms could not be better attested than by this passage. Moreover, in the passage we have already quoted at length (p. 72: 36, 19–37, 2), Philoponus constructed syllogisms in *Barbara*, *Darapti* and *Darii*, all using the copula and all therefore with Prantl's 'perverse' order of the premisses.

We plainly have here a firm school tradition which could rightly claim a genuine understanding of Aristotle's views on the evidence of syllogisms.[32] Prantl, however, in view of the passages he adduces from Apuleius, Martianus Capella and Cassiodorus, states with resignation that "some teachers of logic at least disregarded the order of the premisses, and saw nothing wrong when they had to predicate the minor term of the major" (I, p. 588). But of course the mere transposition of its premisses cannot make any logical difference to a syllogism, still less "turn the major term into the minor". All that happens is that the proposition containing the major (and the middle) term is demoted to second place, as the major term retires from first to second place in the conclusion. Prantl thought that the order of the premisses affected the validity of the syllogism, and saw "something wrong" in their transposition: this only shows that he might well have learned much from those very teachers whom he upbraids so bitterly as the authors of "silly scholastic babblings" (I, p. 588). Apuleius for example actually *discusses* the difference

between his own and the peripatetic order of the premisses: "ordine propositionum et partium commutato sed vi manente ... incipiunt a declarante et ideo a secunda propositione." (l.c. 192.30–193.5). "The order of the propositions and of the terms in them is changed, but their meaning and logical force remain ... They (the Peripatetics) begin their propositions with the predicate and *therefore* (their syllogisms) with (what in my formulation is) the second premiss."

b) *Middle Ages and Modern Times*

Boëthius' formulations were canonized by tradition and the meaning of Aristotle's distinction sank inevitably into oblivion. It would profit little to trace out each of the various attempts to give sense to the distinction between perfect and imperfect syllogisms within the framework of traditional logic. It must suffice to offer a brief description of certain typical and influential theories.

Christian Wolff (*Logica*, § 378) no longer talks of perfect syllogisms but calls the first figure the perfect figure (thus disregarding its Theophrastian moods). The first figure is perfect, he argues, because (a) only first figure syllogisms follow immediately from the so-called 'principle of the syllogism', the "dictum de omni et nullo"[33]; (b) only in the first figure can propositions of all forms, *a*, *e*, *i*, and *o*, be inferred[34]; and (c) only in first figure syllogisms do the premisses contain the reason why the predicate of the conclusion belongs to its subject.[35] Of these points, Aristotle himself offers (a) and (b) as reasons, not for the *perfection* of the first figure, but for its peculiar importance and dignity; (c) is brought up in the *Posterior Analytics*, but there again not to prove the perfection of the first figure but to show that Aristotle's so-called 'apodeictic' arguments, which provide the proofs of the deductive sciences, must always belong to the first figure. It is an epistemological and not a logical point which Aristotle is making: an apodeictic syllogism is always perfect; but not all perfect syllogisms are apodeictic.

J.H. Lambert, an excellent logician, denied any prominence to the traditional first figure in his *Neues Organon* (1764) and, as we have shown, he was quite right to do so. In 1762 Kant revived and refurbished Wolff's points in the unfortunate essay we have already referred to more than once, *Die falsche Spitzfindigkeit der vier syllogistischen Figuren*; indeed, he exceeded Wolff in denying to the last three traditional figures any right

to the status of logical laws, since "they all ... only determine their conclusions with the help of circumlocutions and the interpolation of intermediary arguments". Here Kant is plainly confusing two quite different facts: (a) that some propositions must have certain other propositions added to them before they *entail* a conclusion; and (b) that the fact that some propositions entail a conclusion can only be made *transparent* by a proof. Since (b) and not (a) applies to the arguments of figures II, III, and IV, Kant's reasoning is not sound: it could as well be applied to mathematics to prove that only the axioms, and not the theorems deducible from them, can be recognised as mathematical propositions. The same consideration confutes Prantl's assertion that in syllogisms of the second and third figures "the relationships are improper, since they admit inferences only under certain restrictions and only if the syllogism is reduced to the first figure" (I, p. 271).

Later, the most popular attitude toward the distinction between perfect and imperfect figures (a distinction which traditional syllogistic had obliterated but nevertheless embalmed in its terminology) was an extension of Wolff's reason (c). It complemented the fashion, prevalent in the 19th century and still prevalent today, of bringing Aristotle's logic into close union with his metaphysics.[36] Perfect syllogisms, it was said, are those in which the *ratio cognoscendi* and the *ratio essendi* coincide. Aristotle's own example of such a syllogism runs: "The planets are near; what is near does not twinkle; therefore the planets do not twinkle" (*APst. A* 13, 78a40–b2). Aristotle says that the cause of the planets' not twinkling is the fact that they are near. This is a rather arbitrary assertion; at most it can be only a *part* of the 'cause': certain propositions about the mechanics of the eye, others taken from astronomy would have to be added. No doubt it is reasonable for Aristotle to say that the planets are not near because they do not twinkle but do not twinkle because they are near (78a37–38). But it cannot be inferred from this that the nearness of the planets is the cause of their not twinkling. The reasonable assertion says only that the nearness *belongs* to the cause of the non-twinkling and not vice versa. The nearness, which Aristotle calls the cause, occupies the position of *middle* term in our syllogism. According to another Aristotelian (and equally problematical) theory (*APst. B* 11) the middle term of a syllogism is always the *ratio cognoscendi* of the conclusion. But this is patently a mistaken way of putting it. The cause of our knowledge of

the conclusion is at best the two premisses. It is almost a sophism to displace the two premisses as the *ratio cognoscendi* in favour of the middle term just because the middle term, and it alone, *occurs* in both premisses. And only after this dubious substitution can Aristotle lend any plausibility to the statement that there can, and often does, exist a parallelism between the middle term as containing the cause – the *ratio essendi* – of the fact stated in the conclusion, and the middle term considered as the '*ratio cognoscendi*' of this fact.

Aristotle's theory deserves a thorough investigation.[37] However, it is enough for our purposes if the preceding remarks have shown that the doctrine is in itself highly dubious; that it is not a logical but an epistemological theory; and that it cannot possibly serve as the backcloth to any reasonable discussion of Aristotle's distinction between perfect and imperfect syllogisms. For Aristotle calls *all* assertoric syllogisms in the first figure perfect, while in chapter *A* 13 of the *Posterior Analytics* he says explicitly that the middle term does *not* coincide with the 'real cause' in all arguments of this form. On the other hand, he thinks it very probable, but not certain, that *only* first figure syllogisms have this peculiar property (*APst A* 14, 79a20–22). What really distinguishes this figure is the fact that only its syllogisms, but not all of them, can mediate knowledge of *essence* (ib. 24–9). These properties make the first figure in Aristotle's terminology the κυριώτατον τοῦ ἐπίστασθαι ... σχῆμα (31–32), "the figure most important for science." But all this has nothing to do with the perfection of the figure. It is therefore simply mistaken to see "in the causality of the term alone ... the driving force of the syllogism" (Prantl I, p. 265); or to say that "If the middle term corresponds to the efficient cause, then the syllogism is perfected" (Trendelenburg, *Log. Unt.*[3] II, p. 390) or to think that by designating the first figure syllogisms as perfect Aristotle conveys "the idea that only in the first figure can the cause of knowledge coincide with the real cause" (Überweg, SdL[5], p. 332; cf. 316 and 319, where he asserts that syllogisms in which the middle term is not the real cause are in Aristotle's view 'imperfect').

Traces of this mistaken conception can be found in almost all interpreters of Aristotle's syllogistic who do not start out from mathematical logic. To give an example, the extracts from Maier quoted on p. 40 are obviously inspired by the remarks of Prantl we have just cited. If Aristotle's distinction between perfect and imperfect syllogisms was to be

explained, it seemed necessary, once the traditional formulation of the syllogisms had occluded its proper and straightforward meaning, to conjoin Aristotle's logic, in particular his syllogistic, with his epistemology and metaphysics. This conjunction has made a substantial contribution to the theory, which is still widespread today, that the validity of logical laws is connected in some unspecified but intimate way with certain fundamental facts about the world – facts which ontology and metaphysics investigate in the hope of setting up logic as a 'Logic of Being'.

A far sounder position was that occupied by those logicians and historians of logic who regarded the 'perfection' of first figure syllogisms in traditional syllogistic as a mere dogma, for which, apart from the authority of Aristotle, there was no logical support. Lambert, as we saw, attacked the dogma. Zeller, with nice discretion, reported Aristotle's theory, and renounced any ambition to justify it: "He considers only first figure syllogisms to be perfect, since in them alone, so he believes (sic), is the necessity of the inference at once and of itself apparent."[38] In a footnote he adds: "Here too we may spare ourselves the task of examining Aristotle's opinion."

From Zeller's reserve it is only a short step to Ross' attempt to give a historical *explanation* rather than a logical *justification* of the "tyranny of the first figure" (APPA, p. 33). The starting point of Ross' explanation is Shorey's conjecture (*Classical Philology*, **19**, 1924, pp. 6 sq.) that the seed from which Aristotle's syllogistic grew was a passage in Plato. In the *Phaedo*, 104E–105B, Plato says that the presence (παρουσία) of a specific nature in an individual "brings in with it" (συνεπιφέρει) the genus of which it is a specification. Thus "threeness" brings the generic nature "oddness" into every individual group of three things, and excludes "evenness" from it. It is no doubt true that the syllogisms *Barbara* and *Celarent* can readily be derived from these statements of Plato's; and it is a conjecture well worth considering that Aristotle was, consciously or unconsciously, influenced by the *Phaedo* when he formulated the general principle of first figure syllogisms in *APr. A* 4, 25b32–35. However, Ross continues (APPA, p. 27): "And the fact that only the first figure answers to Plato's formula is the reason why Aristotle puts it in the forefront, describes only first figure arguments as perfect (i.e. self-sufficient) and insists on justifying all others by reduction to that figure": this passage abandons the carefree flights of conjecture, and presents a thesis

which stands in contradiction with the text which it is meant to explain. For "the reason why Aristotle puts it in the forefront" is according to Aristotle's own words (*A* 4, 26b26–33: cf. p. 43) something completely different; and so is the reason why he "describes only first figure arguments as perfect": to pass over Aristotle's own words in favour of "the fertilisation of one brilliant mind by another" (p. 27) is perhaps good psychoanalysis, but it is not philology.

However, besides this 'historical' explanation Ross still has a logical one up his sleeve: "We must remember that Aristotle undertook the study of syllogisms as a stage on the way to the study of scientific method". – Can we "remember" something Aristotle never says? – "Now science is for him the knowledge of why things are as they are". – Perfectly true. – "And the plain fact is that only the first figure can exhibit this." – Aristotle is not so sure of this, as we saw (*APst. A* 14, 79a20–22). – "Take the second figure. If we know that nothing having a certain fundamental nature has a certain property, and that a certain thing has this property, we can infer that it has not that fundamental nature. But it is not because it has that property that it has not that fundamental nature, but the other way about. The premisses supply a *ratio cognoscendi*, but not the *ratio essendi*, of the conclusion" (APPA, p. 33). Ross, like Trendelenburg, Überweg and others, explains the perfection of the first figure syllogisms by reference to the doctrine of the *Posterior Analytics*. We have already said all that is necessary on this point. But Ross' argument is also intrinsically mistaken. He expressly accepts ("it is ... the other way about") that "The thing has not this property *because* it has that fundamental nature"; thus we can construct a syllogism, again in *Cesare* (II), in which the premisses do contain the '*ratio essendi*' of the conclusion: "If we know that nothing having a certain property has a certain fundamental nature, and that a certain thing has that fundamental nature, we can infer that it has not that property". In general, it is plain that we can transform any inference from *Celarent* (I) into *Cesare* or *Camestres* of the second figure. Such a transformation cannot affect the relation between the middle term and the *ratio essendi*. Therefore *if*, as Ross assumes, there are any syllogisms in *Celarent* in which – in the favourite formula – the *ratio cognoscendi* and the *ratio essendi* coincide, then such arguments also occur in the second figure; and if the perfection of a syllogism consists in the coincidence of *ratio essendi* and *ratio cognoscendi*,

then there are perfect syllogisms in the second figure too. And this is clearly opposed to everything Aristotle says.

§ 20. Summary

The distinction Aristotle makes between perfect and imperfect syllogisms classifies arguments according to their evidence. It can be shown to be perfectly reasonable within the context of Aristotle's syllogistic, assuming his idiom "*A* belongs to all *B*", although there is something arbitrary in his classification of *modal* syllogisms. The evidence of first figure arguments can be preserved in the *traditional* formulation (with the copula), if, as logic permits, the premisses are transposed. However, since traditional logic maintained the Aristotelian order of the premisses, the evidence which Aristotle rightly ascribed to first figure syllogisms, and with it the distinction between perfect and imperfect arguments, was obscured. Aristotle's distinction was sometimes misunderstood in antiquity, perhaps even by the first generation of his successors. In particular, it was thought that he did not recognise imperfect syllogisms to be logically *valid*. The commentators too (especially Ammonius and Philoponus) exhibit some uncertainty: they admit 'more and less perfect' arguments; they regard the fact that Aristotle proves a syllogism as the *reason* for its imperfection; and they call not only the *arguments* of the first figure but the *figure* itself perfect. However, a better understanding underlies the practice, which demonstrably survived to Boëthius' day, of writing first figure syllogisms with transposed premisses wherever they were formulated by means of the copula.

This understanding was inevitably destroyed by the tradition which goes back to Boëthius. Aristotle's distinction was then either attacked, or else supported by reference to the *Posterior Analytics* and the erroneous equation of *apodeictic* arguments, which Aristotle there defines, with *perfect* arguments. This in turn gave birth to the theory that Aristotle's syllogistic depends and is founded on the principles of his so-called conceptual metaphysics. This theory has blocked, and still does block, the path to a true understanding of the nature of logic. Ross' attempt to explain Aristotle's distinction historically, by hypothetizing influence from Plato's *Phaedo*, was equally unsuccessful. The interpretation which suggests itself to the commentators (Łukasiewicz and Bocheński) who have

been trained in mathematical logic, that Aristotle so names the perfect syllogisms because he regards them as *axioms*, is superior to any other explanation in so far as it treats the distinction – and treats it seriously – as falling entirely within the realm of logic. However, even this interpretation is not satisfactory, since it does not show how Aristotle could justifiably think that his ascription of 'evidence' to these syllogisms was free from the subjectivity and relativity which seem at first to be essential features of the term. To answer this question the more protracted discussions of the present chapter were necessary. They have shown that the notion of evidence is founded *objectively* on certain formal characteristics belonging to a number of first figure syllogisms.

NOTES

1. Cf. above, pp. 69 sqq.
2. For details, cf. above, pp. 79 sqq.
3. συλλογισμὸς δέ ἐστι λόγος ἐν ᾧ τεθέντων τινῶν ἕτερόν τι τῶν κειμένων ἐξ ἀνάγκης συμβαίνει τῷ ταῦτα εἶναι. λέγω δὲ ‹τὸ?› τῷ ταῦτα εἶναι τὸ διὰ ταῦτα συμβαίνειν, τὸ δὲ διὰ ταῦτα συμβαίνειν τὸ μηδενὸς ἔξωθεν ὅρου προσδεῖν πρὸς τὸ γενέσθαι τὸ ἀναγκαῖον (*A* 1, 24b18–22). τέλειον μὲν οὖν καλῶ συλλογισμὸν τὸν μηδενὸς ἄλλου προσδεόμενον παρὰ τὰ εἰλημμένα πρὸς τὸ φανῆναι τὸ ἀναγκαῖον, ἀτελῆ δὲ τὸν προσδεόμενον ἢ ἑνὸς ἢ πλειόνων, ἃ ἔστι μὲν ἀναγκαῖα διὰ τῶν ὑποκειμένων ὅρων, οὐ μὴν εἴληπται διὰ προτάσεων (22–26).
4. Bocheński too translates (FL, p. 87; HFL, p. 76): "In order to make the necessity of the conclusion apparent."
5. The best text for this meaning of "φανῆναι" is *Met. M* 4, 1079a4: καθ' οὓς τρόπους δείκνυται ὅτι ἔστι τὰ εἴδη, κατ' οὐδένα φαίνεται τούτων.
6. E.g. *APr. A* 14, 33a31: φανερὸς συλλογισμός and *A* 15, 33b35–36: φανερὸν ὅτι (τὸ *A*) καὶ τῷ *Γ* παντὶ ἐνδέχεται· γίνεται δὴ τέλειος συλλογισμός.
7. Ross, APPA, pp. 291–292; Łukasiewicz, AS, p. 43; Bocheński, FL, p. 87; HFL, p. 76.
8. Details above, pp. 81 sq.
9. On this cf. H. Scholz, 'Die Axiomatik der Alten', in *Blätter für deutsche Philosophie* **4** (1930), 259–278, esp. 265 sqq.
10. A detailed account of this will be given in Ch. V.
11. The four figures (see p. 13) then have the form:

$$\text{I}\ \begin{array}{c} PM \\ MS \\ \hline PS \end{array} \qquad \text{II}\ \begin{array}{c} MP \\ MS \\ \hline PS \end{array} \qquad \text{III}\ \begin{array}{c} PM \\ SM \\ \hline PS \end{array} \qquad \text{IV}\ \begin{array}{c} MP \\ SM \\ \hline PS \end{array}$$

12. Cf. PM, *34.1.
13. A system of syllogistic based on these formal foundations has been worked out by P. Lorenzen, 'Die Syllogistik als Relationenmultiplikation', *Archiv f. math. Logik* **3** (1957), 112–116; cf. also Lorenzen, *Formale Logik*, 1958, § 2.

14. Aristotle's *proofs* of the convertibility of *AeB* and *AiB* will be discussed below, 138 sqq.
15. The symbol "$\subseteq$" is introduced by Russell and Whitehead as the sign for inclusion between relations; it is so defined (PM I, 23.01) that $R \subseteq S$ holds if and only if *xSy* holds for all pairs of individuals for which *xRy* holds.
16. Philoponus (113, 20) thinks that Aristotle was content to state the first figure pairs alone because they suffice as examples for pairs of the other figures too: αὐτὸς δὲ μόνον τὴν πρώτην ἔταξε ὡς ἐν παραδείγματι. Ross (APPA, p. 314) writes: "These generalisations are correct, but *A*. has omitted to notice (sic) that *OA* in the second figure and *AO* in the third give a conclusion with *P* as subject". Thus neither of these commentators has seen the logical difference between the cases in the first and those in the other figures.
17. *A* 6, 28b1–4 shows that this is a later correction: ὅταν δὲ ὁ μὲν ᾖ στερητικὸς ὁ δὲ καταφατικός (in the 3rd. figure), ἐὰν μὲν ὁ μείζων γένηται στερητικὸς ἅτερος δε καταφατικός, ἔσται συλλογισμὸς ὅτι τινὶ οὐχ ὑπάρχει τὸ ἄκρον τῷ ἄκρῳ, ἐὰν δ' ἀνάπαλιν, οὐκ ἔσται. – τὸ ἄκρον τῷ ἄκρῳ clearly leaves it open whether the ἐλάττων does not belong to some of the μείζων or vice versa; hence the statement ἐὰν δ' ἀνάπαλιν οὐκ ἔσται, according to ἀεὶ γίνεται συλλογισμὸς τοῦ ἐλάττονος πρὸς τὸ μεῖζον in our passage, would be false. Had Aristotle already seen this point when he came to write *A* 6, he would have had to write τὸ μεῖζον τῷ ἐλάττονι in place of τὸ ἄκρον τῷ ἄκρῳ (28b3).
18. Philoponus (78, 3–9) noticed that the order of the premisses was inverted, and he explained it by observing that the two relational expressions differ only by their σχέσις, that is the spatial ordering of their arguments. In the syllogism constructed with "be contained in" what was before the last term becomes the major term, and what was the minor premiss becomes the major premiss. We cannot tell from this passage whether or not Philoponus noticed that this transposition is necessary to preserve the perfection of the syllogism. A little later, however, he gives two formulations of the syllogism $AaB \& BiC \rightarrow AiC$ with concrete terms: "Animal is said of all men, man of some animate things; Conclusion: animal is said of some animate things"; and: "Some animate things are men, all men are animals. Conclusion, some animate things are animals" (78, 15–17). Here it is plain that the second argument attains evidence because its premisses are transposed.
19. These abbreviations are explained in the Bibliography.
20. John Locke saw this (*Essay concerning Human Understanding*, IV, 17, § 8). Without knowing Aristotle's doctrine of 'perfect' syllogisms, he asked: "... would not the position of the *medius terminus* ... show the agreement or disagreement of the extremes clearer and better, if it were placed in the middle between them? Which might be easily done by transposing the propositions ..." of the traditional first figure. The same idea is found in Leibniz, *Nouveaux Essais*, IV, 17, §§ 4 and 8. – In 1890 E. Schröder hit the nail on the head in his *Vorlesungen über die Algebra der Logik*, I, pp. 173 sq.: "Aristotle's choice of the transposed order (sc. of the premisses) is explained by the fact that he attends not to the extensions but to the contents of the terms; then the arrangement "*c* is a property of *b*, *b* is a property of *a*, ergo *c* is a property of *a*" seems the more natural". E. Scheibe brought this passage to my attention (1968).
21. A. Becker, *Die aristotelische Theorie der Möglichkeitsschlüsse*, Berlin, 1933 (hereafter cited as: ATM). – Ackrill (*Mind* **71** (1962), 112) finds a "certain incoherence" between my analysis of 'absolute' necessity in chapter II and the present adoption

of Becker's interpretation of the modal operators: that may be so – but I had either to produce an analysis of the modal logic of my own (a task which still eludes me), or else follow the best analysis available. Aristotle's modal logic is still a realm of darkness: syllogisms (6) and (4) (above, pp. 62–63), of which (4) is supposed by Aristotle to be *perfect*, are plausible at first sight; but closer inspection shows that they are questionable, indeed evidently invalid, so long as Aristotle's modal operators are read in the traditional way. According to (6), for example, we can construct the following 'syllogism':

> If all humans more than 6 feet tall are men and it is neither necessary nor impossible that all women are more than 6 feet tall, then it is not impossible that all women are men.

(4) will give similar results. Aristotle appears to think that the validity of "If *A*, then *B*" entails the validity of "If possibly *A*, then possibly *B*" (*APr. A* 15, 34a5–7; *Met. Θ* 4, 1047b14–16): that this is specious but false – on the normal interpretation of the operator – can be seen from our deceitful 'syllogism'. On the question, in what sense Aristotle applied the tag "ab esse ad posse valet consequentia" to *implications* cf. the debate between Łukasiewicz, von Wright and Hintikka (above, p. 18).

22. Aristotle defines *P* (*APr. A* 13, 32a18–20) as the modal operator which a proposition or the predicate of a proposition has when neither it nor its negation has the operator *N*. In modern modal logic this operator is called two-sided possibility; one-sided possibility is also found in Aristotle: it is present if a proposition satisfies only the second condition for *P*, i.e. if its negation does not have *N*.
23. *P*1 is the symbol for one-sided possibility ("not impossible").
24. This answers to Aristotle's statement: "It is clear that what is necessary is also actual" (*de Int.* 13, 23a21 sq.). Becker introduces this law as *T* 18 (ATM, p. 15).
25. Philoponus gives this operator the appropriate and pleasing name "half-possibility" (τὸ ἥμισυ τοῦ ἐνδεχομένου *in APr.* 163, 16 et passim).
26. This follows from Becker's *T* 19b: "$p \rightarrow E_1 p$" (ATM, p. 15).
27. Cf. Becker's formula *F* 20 (ATM, p. 15); Bocheński, *Ancient Formal Logic* (hereafter AFL), p. 57.
28. AFL, p. 54; cf. also Bocheński, *La Logique de Théophraste*, p. 59. Bocheński's conjecture that *A* 7 is a later addition and that Aristotle did not have time to systematize his discoveries, is reported and accepted by Łukasiewicz (AS, p. 27).
29. Θεόφραστος δὲ προστίθησιν ἄλλους πέντε ... οὐκέτι τελείους οὐδ' ἀναποδείκτους ὄντας (69, 27–29).
30. Ammonius expressly refers to this difference (*in APr.* 32, 33).
31. Note here (a) that Philoponus, like the other commentators (and Aristotle himself in most cases) inverts the order of the premisses to preserve the evidence of the first figure in spite of the "*B* is *A*" formulation; (b) that Philoponus formulates his syllogisms no longer as propositions but as *rules*, the predominant practice since Apuleius and Boëthius (ob. 525 AD).
32. Sextus Empiricus too (3rd century AD) offers as a "Peripatetic syllogism" an argument in *Barbara* with transposed premisses: "The just is beautiful; the beautiful is good; therefore the just is good" (*Pyrrh. Hyp.* II, 163).
33. *Philosophia rationalis sive Logica*[3] (1744), § 388, p. 317: "Quoniam syllogismi primae figurae non sunt nisi applicatio Dicti de omnibus et nullo, istud autem

Dictum per se evidens est, ... quivis in prima figura syllogismus suam fert evidentiam." So too in J. Jungius, *Logica Hamburgensis* (1638), p. 139 of R. Meyer's new edition, Hamburg, 1957.

34. ib. § 400, p. 327: "Figura perfecta dicitur, in qua omnes propositiones inferri possunt, imperfecta contra, in qua non omnes inferre licet."
35. ib. § 393, p. 322: "Ceterum apparet in his modis syllogismorum ... medium terminum non continere rationem, unde intelligitur, cur praedicatum conveniat subjecto."
36. Examples could be multiplied. Prantl (I, p. 348) laments the process, completed in late antiquity, of "releasing logic from the bonds with which, in Aristotle, it had in general been bound to philosophy"; he talks (I, p. 402) of the "Platonic and Aristotelian principle of a logic closely conjoined with philosophy"; he stresses (I, p. 136) "the real metaphysical side of Aristotle's logic"; he speaks (I, p. 104) of "the inseparable unity of logic and metaphysics" in Aristotle.

    Trendelenburg, *Logische Untersuchungen*³ (1870), I, p. 32, writes: "Aristotle's fine discussion showing that the middle term of a true syllogism corresponds to the ground of the fact, has been pushed to one side and ignored by formal logic"; ib.: "Kant blotted out the last traces of its metaphysical origin".

    Maier (SdA II, 2, p. 85): "Inference-theory can never dispense with its epistemological and metaphysical foundations". Further (p. 386): "The metaphysical background of Aristotle's original logic, to which it owes its real synthetic power, has been forgotten since the Stoic plague". Solmsen follows Maier (*Die Entwicklung der aristotelischen Logik und Rhetorik*, 1929, p. 54): "Thus the reduction of all the other figures to the first ... in fact means precisely their reduction to the ontologically rational line of terms".

    Another recent scholar to state and emphasize this interpretation of Aristotle's logic is N. Hartmann (*Grundzüge einer Metaphysik der Erkenntnis*, 2nd ed., 1925, p. 22): "There can be no doubt that Aristotle thought of logic from an ontological point of view, and that he meant it to prepare the way for 'first philosophy' or 'the science of being as being'".
37. Cf. the discussion of the passage in J. König, 'Bemerkungen über den Begriff der Ursache', in *Das Problem der Gesetzlichkeit*, Hamburg, 1949, esp. pp. 109–117.
38. E. Zeller, *Die Philosophie der Griechen* II, 2², 1862, p. 166.

THE FIGURES

§ 21. The Chief Difficulties

At the beginning of the book (§ 1, p. 1) we noted briefly that traditional syllogistic divides the valid syllogisms into four 'figures', and that these figures are formally distinguished from one another by the position of the middle term (the term which occurs in both premisses but not in the conclusion). In the first and fourth figures the middle term stands chiastically; in the second it stands at the end of both premisses, and in the third at the beginning of both. The first and fourth figures are distinguished by the fact that in the first figure the middle term stands at the beginning of the first and at the end of the second premiss, in the fourth at the end of the first and at the beginning of the second premiss.

This classification is relative to the traditional formulation of premisses and conclusion, which, as we have shown, is not the same as Aristotle's. It is clear at once that Aristotle's way of formulating the propositions of a syllogism (with "be said of" or "belong to" instead of the copula, cf. § 5) inverts the order of the terms inside the premisses, so that in *Aristotle* the middle term occurs at the beginning of both premisses in the second figure and at the end of both in the third. However, we have seen that Aristotle does not adhere firmly to this order and that sometimes, instead of "A is said of B" and "A belongs to B" he says "Of B A is said" and "To B A belongs" (examples on p. 9). Hence the position of the terms in the premisses cannot for Aristotle be in itself a sufficient criterion for defining the figures: for this position is *variable within* the figures. This suggested that not the mere *position* of the middle term, but its grammatical *function* in the premisses should be regarded as the criterion for Aristotle's classification: the second figure would then be that in which the middle term is twice predicate, the third that in which it is twice subject. And the first figure could be defined as that in which the middle term is subject of the major premiss and predicate of the minor, whereas in the fourth it is predicate of the major and subject of the minor. This

interpretation has been generally favoured. We shall see, however, that Aristotle, like his successors, introduces a 'formalist' definition of the figures and of the major and minor terms, which, like all formalist definitions, is relative to certain designated 'standard formulations'. It is plain that here as in the other peculiarities of Aristotle's theory which we have already investigated, we are bound to import extraneous difficulties into the text if we disregard Aristotle's own standard formulations and attempt instead to relate his definitions to the formulations of *traditional* logic. And then, just as in the question of what Aristotle meant by his distinction between perfect and imperfect syllogisms, we can *only* explain the idiosyncrasies of Aristotle's logic by adducing adventitious assumptions about its 'real nature'. This is what the commentators have done; and, disparate though their assumptions may be, they have this in common: they are all peculiarly adapted to make an originally clear point artificially obscure – a consequence which is sometimes unintentionally but always appositely expressed by the assertion that Aristotle's logic cannot be understood without certain "metaphysical foundations" (cf. Maier, SdA II, 2, pp. 84, 255–260).

The conjecture that Aristotle must have divided his syllogisms into figures by some process other than the traditional one was supported by the fact, mentioned in § 1, that he only discusses the first three figures of traditional logic, leaving the fourth unconsidered. This was of necessity the more astonishing in view of the (often ignored) fact that he recognised the valid moods of this figure as valid syllogisms (*APr A* 7, 29a19–27: *Fesapo* and *Fresison*; *APr. B* 1, 53a9–14: *Bamalip*, *Calemes* and *Dimatis*; cf. § 14, pp. 55 sq.). It was a short step to suppose that Aristotle's classification obeyed some *peculiar* principle; and hence Maier (SdA II, 1, p. 48, n. 1) could say: "What principle the three Aristotelian figures are based on is one of the most difficult questions which Aristotle's logic poses his interpreters." The long chapter (l.c. pp. 47–71) which Maier devotes to this problem was rightly called by Łukasiewicz "one of the most obscure chapters of his laborious but unfortunate book" (AS, p. 36). In it Maier discusses in detail two basic interpretations, one of which (Trendelenburg, *Log. Unt.*[3] II, pp. 342 sqq.) sees as the principle of Aristotle's classification the relationship between the *extensions* of the middle term and of the two outer terms; the other (Überweg, SdL[5], § 103, pp. 326–345), pointing out the logical difficulties in Trendelenburg's position,

supposes rather that Aristotle let himself be guided by the *function* of the middle term as subject or predicate of the premisses. Maier follows Trendelenburg, going beyond his position. Of the most recent commentators, Ross too follows Trendelenburg in all essentials, while Łukasiewicz is in substantial agreement with Überweg.

The earlier authors, with the exception of Überweg, possessed no clear appreciation of the logical aspects of the situation. They nevertheless tried to interpret Aristotle's text, oblivious of the danger that they might thus, without any compelling reason, burden Aristotle with the grossest logical errors. This was an after-effect of Prantl's fateful dogma: he ordained that a logical doctrine was correct just because it stemmed from Aristotle, and that a thesis which Aristotle did not propound was thereby shown to be not only unimportant but simply false (Prantl I, pp. 272, 488). Łukasiewicz, on the other hand, possessed a perfect technical understanding of the problems treated by Aristotle. But the level of his discussion, it seems to me, is in places so high above Aristotle's that his contentions cannot provide a precise understanding of Aristotle's text. In a similar vein, Łukasiewicz thinks that the lack of the fourth figure in Aristotle, which earlier commentators stress so heavily, is due merely to negligence.[1] To explain the omission, he conjectures, like Bocheński[2], that Aristotle discovered the validity of the fourth figure moods late in life and had no time to work out his discovery systematically.

The interpretation which I shall propose in the following pages can claim these points in its favour: (a) it is faithful to Aristotle's text; and (b) it renders the omission of the fourth figure an explicable *consequence* of Aristotle's *definitions*. There may be objections to it which I have not seen. The *correctness* of an interpretation cannot be *proved*: exegesis is scientific only in that it can be *dis*proved if it is *in*correct: the philosophical interpretation of a classical text is 'definite'[3] inasmuch as it is refutable.

In the following pages Aristotle's texts on our question will first be produced and discussed; then the most important of the earlier interpretations will be briefly reported and examined; and finally I shall summarise the interpretation which we shall have won from the text and tested by comparison with previous theories.

§ 22. The Text: Definition of the First Figure

I give the relevant sections of Book *A* of the *Prior Analytics* in translation; the excerpts are complete and follow the order of the text.

1) *A* 4, 25b32–37[4]:

(a) "When three terms (ὅροι) are so related to one another that the last is in the middle (as in a) whole, and the middle is or is not in the first (as in a) whole, then there is necessarily a perfect syllogism of the outer terms.

(b) I call middle (the term) which both is itself (contained) in another (term) and another (term) is (contained) in it, which is also middle by position (θέσει).

(c) I call outer (the term) which is itself (contained) in another and (that) in which another is (contained)."

Here, as elsewhere at the beginning of a discussion (cf. p. 45), Aristotle's language is intentionally lax and in some places possibly misleading: the phrase "τὸ ἔσχατον ἐν ὅλῳ εἶναι τῷ μέσῳ" (b33) – formally "*C* is in the whole *B*" – gives the impression that *C* ought to be *at least* coextensive with *B*[5], although Aristotle obviously means that *C* must be *at most* coextensive with *B*. The usual translation, which we have followed, is therefore correct: "The last is in the middle *as in a* whole". We can also refer to the definition which Aristotle has given a few lines before (*A* 1, 24b26–28): ""The one is in the whole other" and "the other is said of all the one" are the same"[6]. That is, "*A* is in the whole *B*" means for Aristotle exactly the same as "*B* is said of all *A*".

According to the same passage (24b30), the expression "ἐν ὅλῳ τῷ πρώτῳ μὴ εἶναι" can always be replaced by the equivalent "τὸ πρῶτον κατὰ μηδενὸς (τοῦ μέσου) λέγεσθαι". Sentence (a) of our text can therefore be rewritten on the basis of this definition as follows: "When three terms are so related to one another that the first is said of all or none of the middle, and the middle is said of all the last, then there is necessarily a perfect syllogism of the outer terms." In this version the relation between the terms is expressed by "be said of" instead of "be contained in". Furthermore, the original order of the premisses is changed. This is

necessary because (a) asserts that a *perfect* syllogism ensues (cf. § 15, esp. p. 58): unless the premisses are transposed the syllogisms in question would not be perfect. Philoponus remarks on this: "Note how he (Aristotle) pointed out to us the peculiarity of the first figure, by saying 'The last is in the middle as in a whole' instead of 'The middle is said of all the last'".[7] And later: "'of all' and 'in the whole' are the same and differ only in *form* – so that when we make syllogisms with 'of all', the first term is the major and the first premiss the major; but when (we make syllogisms) with 'in the whole', the last term (is the major) and the last premiss (the major)".[8]

The need to transpose the premisses if the syllogisms are to be perfect, together with these remarks of Philoponus', give us all we need to resolve a mild paradox presented by sentence (a) – the odd fact that Aristotle here introduces the three terms as "last", "middle" and "first", although in this sentence – their first occurrence – they appear in exactly the opposite order. Some have wanted to take this paradox as an argument in favour of the view that Aristotle meant these titles to indicate, not the order of the terms in any *propositions*, but their order in a *pyramid of terms* ascending from particular to universal. This is held by Waitz (I, pp. 379 sqq.) and Prantl – who in his rendering of our sentence (a) replaces the words "last term" by "lower term" and "first term" by "higher term", without any hint that he has changed the text; he retains the expression "middle term", but obviously with a different meaning. Just before this he asserts, following Waitz, that the ἔσχατος ὅρος is called "the outermost term because of its relationship to the indivisible particular" (I, 271).

Such assumptions, unfounded in the text, need not (and therefore *should* not) be made to resolve the paradox in question: in sentence (a), the *only* place in *A* 1–7 in which a syllogism is expressed by means of the constant "... be contained in ...", Aristotle calls the term which here occurs first the last term because it appears last in his *standard* formulation (which soon follows). We have already constructed this standard formulation by applying Aristotle's definitions to sentence (a); in fact it follows in the text immediately after our passage: "If the *A* is said of all the *B* and the *B* is said of all the *C*, then the *A* must be said of all the *C*" (b37–39). This is the standard formulation; in it *C* is the last term; and therefore, in the equivalent but formally different sentence (a), Aristotle

also calls C the last term, although there it is the first. Similarly, we find a little later: "If the first follows all the middle and the middle belongs to none of the last, there will not be a syllogism of the outer terms" (26a2–4). Here the relational expressions "follow" and "belong to" (cf. § 4, p. 9) correspond, as regards the order of the terms, to "be said of". On the other hand the copula ("A is B" or "All A is B") is in this respect formally equivalent to "be contained in". The two groups of expressions are *converse* to one another: if A is contained in B or all A is B, then B is said of all A, B belongs to all A, and B follows all A.

A comment which will have important consequences must be made on the apodosis of sentence (a), "then there is necessarily a perfect syllogism of the outer terms". The formula "there is a syllogism of the outer terms" would leave it open which of the two outer terms is to be predicated of the other in the conclusion. But the additional stipulation that the syllogism in question is to be *perfect* singles out one of the two possibilities: the last chapter showed that the term named first in the standard formulation (τὸ πρῶτον) must be the *predicate* of the conclusion, if the argument is to be in Aristotle's sense perfect. Hence, in the formulation of *Barbara* (and *Celarent*) (25b37–26a2) the proposition AaC (AeC) is stated as 'the' conclusion, although from the given premisses CiA (CeA) would follow just as well. And since the first figure syllogisms *must* always be perfect, Aristotle laid it down that, in the other figures too, the conclusion of a valid inference should have as its predicate the term which, not counting the middle term, is named first in the premisses. This convention alone explains how Aristotle can say that in the third figure no syllogism with an *a*-proposition as major and an *e*-proposition as minor premiss is possible (*A* 6, 28b1–4); while in *A* 7, 29a22–23 he explicitly (and correctly) asserts that in these cases a syllogism with CoA as conclusion can be constructed. In *A* 4–6 the distinction between valid, i.e. concludent, and inconcludent pairs of premisses is founded on the assumption that in a valid syllogism the outer term *first*-named in the premisses must be the predicate of the conclusion. Then in *A* 7 Aristotle appends the cases in which the conclusion has the form *converse* to this. Such syllogisms are treated in *A* 4–6 as simply invalid.

This fact, and this alone, also explains a locution which seems at first to be a flat contradiction (*A* 7, 29a19–23): "when there is no syllogism ... then there is always a syllogism of the minor term with regard to the

major"[9], that is, a syllogism in the conclusion of which the minor term is predicate. Aristotle's statement "if there is no syllogism ... then there is always a syllogism" escapes the charge of absurdity[10] only if in the *antecedent* he is using the word "syllogism" in the sense implicitly defined by his *practice* in the preceding chapters *A* 4–6 (where the major term is predicate of the conclusion), and if in the *consequent* he is using the word in the sense sanctioned by *A* 7, in which there are no restrictive stipulations governing the function of the major term in the conclusion (Cf. p. 85, n. 17).

Alexander (109–110, esp. 109, 10–12[11]) offers a comparable solution to this paradoxical assertion. He says that, although the additional pairs of premisses in question are "inconcludent, since they cannot prove what was (originally) proposed", it is nevertheless possible "to infer and prove something *else* from them". This explanation is inconsistent both with Aristotle's assertion that "nothing whatsoever" can be inferred from the pairs he has shown to be 'inconcludent' (*A* 4, 26a4), and with his definition of the syllogism (*A* 1, 24b18–20) which leaves it quite open what sort of proposition it must be which "follows from the premisses with necessity" and requires only that it must be a proposition different from the premisses.[12] However, since we are dealing here with an inconsistency in Aristotle's use of the word "syllogism", every rescue operation must either come into conflict with Aristotle's doctrines or else break the laws of logic.

Philoponus, among others, took the latter course. He relates Aristotle's expression to a distinction between pairs of premisses which are "totally useless" (παντελῶς ἄχρηστοι: 112, 24) and those which allow something to be inferred "after conversion of the premisses", and then offers the two classes as subsets of the set of all inconcludent premiss-pairs. But it is *logically* false to suppose that after conversion of the premisses something can be inferred from them which could not have been inferred before. The strength of a proposition can never be increased by a valid conversion, although it can sometimes – as in the 'impure' conversion from *AaB* to *BiA* – be diminished. Conversion of the premisses can make the implication between them and the conclusion *clearer*: it cannot *establish* such a relation. The inspiration behind Philoponus' explanation is clearly to be found in one of Aristotle's less happy formulations: Aristotle himself says (*A* 7, 29a23–27): "e.g. if the *A* belongs to all the *B* or to some,

and the *B* belongs to no *C*. For, ἀντιστρεφομένων τῶν προτάσεων, the *C* must not belong to some *A*. Similarly in the other figures too: there is always a syllogism διὰ τῆς ἀντιστροφῆς".[13] How are we to translate ἀντιστρεφομένων and διὰ τῆς ἀντιστροφῆς? Philoponus took them as *conditional* and *instrumental*: "then after conversion of the premisses the *C* must not belong to some *A*" (cf. 112.30–113.2: εἰ δὲ ἀντιστρέψωμεν ...); and "there is always a syllogism *by means* of conversion". It is natural enough to take the words in this way: but it would convict Aristotle of an elementary logical error, and would hardly be compatible with the immediately preceding sentence which states simply that a syllogism with these two premisses and *CoA* as conclusion is valid. We must therefore suppose that ἀντιστρεφομένων gives the *ground* for the assertion that in the case of such premisses *CoA* necessarily follows. Similarly, διὰ τῆς ἀντιστροφῆς must be taken *causally*. This offends against the grammar books, but not against Aristotle's usage (cf. Bonitz, *Index Aristotelicus* 177a38 sqq.). In this case Aristotle would merely be saying that *CoA* follows from *AaB* and *BeC*, and that the truth of this assertion can easily be ascertained since both premisses are *convertible*. *CeB* and *BiA* yield, by the perfect syllogism *Ferio*, *CoA*.[14] Aristotle is not maintaining that these syllogisms are at first invalid arguments and only become valid when their premisses are converted: he is saying two different things in one sentence: (a) that these syllogisms are valid; and (b) that by conversion of their premisses they can be reduced to a perfect syllogism (here *Ferio*) and thus proved, in the way common to all other imperfect arguments. However, his way of saying this is so open to misunderstanding that we cannot help feeling the translation which has been general since Philoponus to be the correct one – unless we perceive the logical absurdity with which it burdens Aristotle.

Waitz (I, p. 392) offers a striking paraphrase: "etiamsi autem ex conjunctione propositionum ἀνομοιοσχημόνων[15] non fit unus eorum *syllogismorum* quos exposuimus, tamen ex ea elici potest *conclusio*, in qua terminus minor de majore praedicatur". He thus removes the *paradox* by assuming an *equivocation* in the text: "συλλογισμός", according to Waitz, must mean "argument" in 29a20 and "conclusion" in a23. It is true that in Aristotle, "συλλογισμός" does sometimes mean the same as "συμπέρασμα", that is "conclusion" (evidence in Bonitz, *Index*, 712a5–10); but this is only because every argument which has a conclusion is a syllogism,

and vice versa. Waitz tries to distinguish between οὐκ ἔσται συλλογισμός and οὐδὲν γὰρ ἀναγκαῖον συμβαίνει; the former phrase indicates only that none of the syllogisms described in *A* 4–6 can result, the latter the utter impossibility of any syllogistic consequence at all. But this attempt founders on the counter-instance at *A* 4, 26a2–5: there the assertion οὐκ ἔσται συλλογισμὸς τῶν ἄκρων is *explained* by the immediately following οὐδὲν ἀναγκαῖον συμβαίνει; and the case in question is precisely *AaB* & *BeC* from which according to *A* 7 *CoA follows*. Prantl (I, p. 277) repeats Philoponus' logical error; Maier follows Waitz' interpretation but embellishes it with a few logical curiosities – as when he speaks of the moods treated in *A* 7 as "second class arguments", which "being imperfect and powerless in themselves stand in need of proof" (SdA II, 1, p. 98). But what could a proof possibly demonstrate other than the 'syllogistic power' of these moods? An argument which has no 'syllogistic power' is a false argument: and no proof can conjure falsity into truth. The very title of this section, "Arguments from syllogistically useless combinations", reveals the logical calibre of Maier's lucubrations. This, it may be remarked, is a good example of how even attentive readers of Aristotle's text can go astray if they have no logical training and lack the Ariadne's thread which it alone can provide.

These remarks should have made sentence (a) of our extract (p. 91) sufficiently clear. Sentences (b) and (c) give supplementary explanations of the terms appearing in (a) – this is a practice we have already met (e.g. in the definition of the syllogism: cf. § 12, pp. 44 sq.) Unfortunately, the definition of "middle term" is almost verbally identical in the Greek text with that of "outer term"; if we took the definitions at their word, the middle term would *also* be an *outer* term. The awkwardness is plainly due to Aristotle's desire to set the two outer terms together in contrast with the middle term. For this reason he unites the properties of both outer terms in one definition. Philoponus observed this infelicity (73, 15–18) and proposed to remove it by adding μόνως before καὶ (b37) and at the end of sentence (c). καὶ would of course then have to be emended to ἢ. Aristotle clearly means to designate as the middle the term which contains one of the outer terms and is itself contained in the other. The outer terms are, consequently, (i) the term which contains in itself both the middle term and the other outer term, and (ii) the term which is contained in the middle term and in the other outer term.

There remains the relative clause of (b): ὃ καὶ τῇ θέσει γίνεται μέσον. This sentence has been variously understood by its interpreters, and is of importance for the understanding of Aristotle's theory of the syllogistic figures. We shall for the moment put it aside and discuss it in detail later, together with the corresponding formulations in the descriptions of the second and third figures.

We must next observe that although Aristotle uses the expressions "outer term" and "middle term" in all the figures, he offers *new definitions* of them for each of the figures (*A* 5, 26b36–37; and *A* 6, 28a12–13). Thus Aristotle himself recognised that the present definition only holds in the case of the first figure. However the definition must be *universally* valid within the first figure; and this assertion of Aristotle's is false. First, as earlier writers (e.g. Überweg, SdL[5], pp. 332 sq.) have pointed out, the definition in terms of the constant "be contained in" can only be correctly applied to syllogisms in *Barbara*. For, according to Aristotle himself (*A* 1, 24b26–28), we can only talk of a term *A*'s being contained in a term *B* by virtue of a previous judgment of the form "All *A* is *B*" or "*B* belongs to all *A*". On the other hand, a judgment of the form *BiA* ("*B* belongs to some *A*"), *BeA* or *BoA* gives no clue at all as to the relative extensions of *A* and *B*. The definitions of the outer and middle terms cannot therefore be applied to *Celarent*, *Darii* and *Ferio* of the first figure. Moreover, Łukasiewicz (AS, pp. 28–30) has argued that they do not even fit *all* syllogisms in *Barbara*. They are satisfied only by the terms of those syllogisms in *Barbara* whose premisses are *true* propositions. But the validity of a syllogism is independent of the truth of the propositions which compose it. This is not only a logical truth, but an explicit thesis of Aristotle's (*APr. B* 2–4).[16] Besides, we have already seen that the operator ἀνάγκη in Aristotle's formulation of, say, *Barbara* corresponds to a universal quantifier over the terms *A*, *B*, and *C* which occur in the syllogism (§ 7, pp. 26–28). *Barbara* would not be a logical law if it were not true whatever terms replace its variables *A*, *B*, and *C*. Thus the argument obtained by inserting concrete terms in *Barbara*, "If man is said of all animals and animal is said of all Eskimos, then man is said of all Eskimos", is a *true* proposition, even though its first premiss is false. And the middle term "animal" is certainly not contained in either of the outer terms but contains them both. Aristotle's definition would better fit "man", the substitute for *A*; but "man" is not thereby made the middle

term. For the middle term must, according to Aristotle, occur in both the premisses (*APr. A* 32, 47a39).[17] However, we can easily explain how Aristotle came by his mistaken definition. He constructed some syllogisms in *Barbara* with true premisses and found that in these concrete arguments the extensions of the terms were related in a certain way; his definition then required that they be related in this way in *all* syllogisms of the first figure. Thus his definition grew in two steps by incorrect generalisation of correct observations about certain concrete arguments in *Barbara*: what holds for arguments in *Barbara* with concrete terms and true premisses, was supposed to hold for all syllogisms in *Barbara* – and then for all first figure syllogisms.

However, Aristotle's most important mistake is that, although he formulates his syllogisms with *variables*, he introduces definitions which only make sense when applied to arguments with concrete terms. Clearly we can only compare the extensions of concrete terms, not of variables. The failure to distinguish between logical laws and the concrete arguments which are produced by the substitution of concrete terms for variables in these laws is a short-coming of Aristotle's logic; it receives expression in the fact that Aristotle indiscriminately calls both logical laws and concrete arguments "syllogisms" (compare e.g., *APr. A* 5, 27b3 and *A* 6, 28a27 with *A* 34, 48a13 and *APst. A* 13, 78a36).

In the texts we have so far discussed Aristotle distinguishes middle from outer terms by means of an (incorrect) definition. (A definition as such is of course neither true nor false, for it is only an explanation of how the author wishes a word to be understood. The expression "incorrect definition" is meant only as an abbreviated way of saying that Aristotle's text treats as satisfying his definition objects which in fact do *not* satisfy it.) The outer terms, however, have not yet been explicitly distinguished from one another; they are called, answering to their order in the standard formulation, ὁ πρῶτος ὅρος and ὁ ἔσχατος ὅρος or τὸ πρῶτον ἄκρον and τὸ ἔσχατον ἄκρον (cf. *A* 4, 26a2; *A* 36, 48a40, b1; *B* 8, 59b2; *B* 23, 68b34). Since these expressions are relative to the standard formulation of the first figure, they must be used only in the representation of first figure syllogisms, if they are not to cause confusion. However, in *A* 4 Aristotle offers another way of distinguishing between the outer terms, which, in the first figure at least, is meant to be independent of their order in his standard formulation; he is discussing *Darii*

and *Ferio* (*A* 4, 26a17–30): "I call greater (major) the outer term in which the middle is contained and less (minor) that which is under the middle term" (a21–23).[18] The expression "*A* is under *B*" is never defined by Aristotle; but he often uses it (e.g. *A* 9, 30a40; *A* 11, 31a30; b17) to express the relation *BiA* – and in *A* 10, 30b13 to express *BaA*. It is easy to see why in our passage Aristotle drew on this undefined expression (which often denotes logical subordination in his writings: cf. Bonitz, *Index*, 795a34–50) in order to explain the relationship between middle and minor term: he is discussing *Darii* and *Ferio*, and in both these moods the second premiss is *particular*; the idiom "*A* is contained in *B*", however, had been introduced expressly as a way of formulating a *universal* judgment. The change of expression shows clearly that Aristotle saw the difficulties which faced his attempt to define the terms of the first figure by their relative *extensions*, difficulties raised at once by the fact that two of the first figure moods contain *particular* premisses. The word "ὑπό", which here suddenly comes upon us in place of the expected "ἐν" and is not, like "ἐν τινὶ εἶναι", *defined*, bobs on the surface of the text like a buoy marking shallows. It is remarkable that no interpreter, from Alexander to Łukasiewicz, has taken exception to the change of expression.

Of course, it is easily shown that this definition too is universally applicable only to the terms of an argument in *Barbara* with concrete terms and true premisses; a counter-example is provided by the following concrete argument: "If ruminant belongs to all cattle and cattle belongs to some animals, then ruminant belongs to some animals." This is an argument in *Darii* (I). By Aristotle's definition, "ruminant" is the major term, "animal" the minor. And since Aristotle asserts that in the second premiss (*BiC*) the minor is "under the middle term", he must regard the minor as *subordinate* to the major in the conclusion too. But in fact the major is obviously subordinate to the minor – and that in a syllogism with true premisses. Aristotle's adoption of the expressions "*major* term" and "*minor* term" depends on the assumption that a term which in his idiom is 'under' another must also be subordinate to it. But this is by no means the case. Aristotle's terminology is therefore misleading (cf. Łukasiewicz, AS, p. 30).

In these circumstances it is astonishing that Aristotle uses "μεῖζον" and "ἔλαττον" – words fraught with misleading associations – as the *standard* expressions for the outer terms in the second and third figures

too; for although in his discussion of these figures he never explicitly *says* that a definition of the terms in them by means of their relative extensions is impossible, in fact, as we shall see, he follows a different path to their definition and never attempts to say anything about their extensions. Surely the obvious course would have been to keep to the expressions "πρῶτον" and "ἔσχατον" which, as we saw, say nothing whatever (*pace* Prantl I, p. 271) about relative extensions? This, however, would have led to a certain inconcinnity: for these expressions would clash headlong with Aristotle's definitions of the terms in the second and third figures (which, it will appear, are based on their *order* in certain standard formulations). He would have had to define "first term", for example, as "second term of the second figure", "last term" as "second term of the third figure". So in fact it was more convenient to keep "major" and "minor" as *names* for the outer terms, and to take care that their *definitions* cancelled the extensional implications, inevitably suggested by the words, which caused difficulties even in the first figure.

§ 23. The Text: Definitions of the Second and Third Figures

At the beginning of chapter *A* 5 of the *Prior Analytics* Aristotle gives a definition of the second figure: "(a) But when the same (term) belongs to all the one and none of the other, or to all of both or to none of both, I call such a figure the second. (b) In this figure (I call) middle that (term) which is said of both (the other terms) and outer those (terms) of which it is said (c). (I call) greater outer that (term) which lies next to the middle, less that which is further away from the middle. (d) But the middle (term in this figure) is outside the outer (terms) and first in position."[19]

Sentence (a) traces out certain combinations of premisses, namely: *BeA* & *BeC*, *BeA* & *BaC*, *BaA* & *BaC*, *BeA* & *BeC*. These pairs of *premisses* serve as examples for the second *inference*-figure which Aristotle is about to discuss. The last two are not premisses of valid syllogisms; while on the other hand the premisses of *Festino* and *Baroco* are missing. Aristotle is clearly only interested in combinations of *universal* premisses, and in an exhaustive list of these. He only wants to note in fairly loose terms some obvious examples of premisses of this form; closer examination, testing which of these combinations can yield syllogisms, is left to later

inquiry. In the examples which Aristotle proffers no conclusion is ever formulated. Nevertheless Aristotle calls these combinations of premisses examples of the second figure. This does not harmonise well with our ingrained assumption that the *Prior Analytics* investigates *syllogisms* – their formal differences, their validity or invalidity. The better, however, does it fit Aristotle's actual practice: the objects of the inquiry in *A* 4–6 are, almost without exception, not complete syllogisms but pairs of propositions which have a term in common (συζυγίαι, as the commentators call them). Aristotle shows how these pairs differ formally from each other, and asks *whether* they yield any conclusion, and if so, *what*. We shall have more to say about this later; it will be evident at once that Aristotle's method is more *convenient*: there are (in three figures) 48 possible pairs of propositions – there are 48 times 4, or 192, possible arguments.

Aristotle goes on to say (b35) that he calls *such* a schema (τὸ τοιοῦτον σχῆμα) – not this schema (τοῦτο τὸ σχῆμα) – the second figure. We can understand this expression if we remember that Aristotle did not have any fixed term for "mood". Hence, instead of "such syllogisms are moods of the second figure", he says "such a figure is the second". In exactly the same way, he calls the moods *Celarent* (*A* 5, 27a12) and *Ferio* (*A* 5, 27a36) simply "the first figure" (τὸ πρῶτον σχῆμα). The expression "πτῶσις" and "τρόπος τοῦ συλλογισμοῦ", the latter of which is the accepted phrase for "mood" in the commentators, do not appear at all in *APr. A* 4–6, and in the other (later?) parts of the *Organon* they occur only rarely in connexion with the syllogism and then without any fixed technical sense. Thus in *APr. A* 28 "τρόπος" stands for "*figure*" (45a4), but at *A* 26, 43a10 it is used, like "πτῶσις" (42b30), for "*mood*". "Figure" is thus for Aristotle primarily a formal property of *syllogisms*. On the other hand he also says that "all syllogisms *in* the first figure are perfect" (*A* 4, 26b29). Thus "σχῆμα" in Aristotle names both a *property* of the syllogism itself and the *class* to which the different syllogisms belong by virtue of this property.

Sentence (b) defines "middle" and "outer term", not, as in the first figure, by means of their relative extensions, but by reference to their grammatical *function* as subject or predicate in the two premisses. The middle term in the second figure is defined simply and solely by the fact that it is the predicate of both premisses. The outer terms are then plainly

the terms which occur as subjects in the two premisses. How then are the outer terms to be distinguished *from one another*? Both are subjects of the middle term and so do not differ at all with regard to their function. There is no path left but to define them according to the order of their occurrence; and to this end, since the order is to some extent arbitrary, a *standard formulation* must be chosen. The standard formulation of syllogisms in the second figure follows immediately on the general discussion (27a5 sq.): "Let *M* be said of no *N* and of all *X*"; and again: "If the *M* belongs to every *N* and to no *X* (then the *N* will belong to no *X*)" (a9–10) etc. Here *M* stands for "αὐτό", *N* for "τῷ μέν", and *X* for "τῷ δέ" in the general schema of sentence (a) (26b34).

Aristotle now defines in the simplest possible way the so-called major as the term which *in the standard formulation* "immediately follows" the middle term, and the minor as the term which in the same formulation is "further away from it". It is clear that all previous commentators have been hampered by the prejudice that such a procedure is not philosophical enough: nothing else can explain the recurrent attempt to take what are obviously *spatial* terms as metaphors for some (really intended) 'inner relationship' – 'functional identity' or 'similarity of extension'. In the literature on the subject our proposed interpretation has more than once been at least mooted as a possible solution of these interminable difficulties;[20] but it has not been able to make headway against more profound explanations. The main reason for this has been ignorance of the difficulties which Aristotle had to face in his attempt to give definitions, and the consequent inability to realize how welcome he must have found the possibility of giving 'formalistic' definitions based on a standard formulation. On our interpretation, sentence (d) follows (c) without a break: in this standard formulation the middle term does stand (in contrast to the first figure) "outside the outer terms" – in fact it assumes (in contrast to the third figure) the first place in the argument. Consistently with this, where Aristotle wants to prove that a given pair of premisses yields no conclusion he introduces the terms in the second figure in the order *M*, *N*, *X* (middle, major, minor) (e.g. 27a19–20), while in the case of the first figure he never abandons the order major, middle, minor (e.g. *A* 4, 26a8–9 et passim).

In sentence (b) Aristotle defines "middle" and "outer term" by their distinct grammatical functions as subject and predicate of the premisses.

The difference between major and minor term however, cannot be so determined. *Therefore* Aristotle introduces a *standard formulation* and defines "major" and "minor term" by their order of occurrence in it. An additional way of defining the middle term is now open: it is unequivocally determined by its position in the same formulation. The appended clause, ὃ καὶ τῇ θέσει γίνεται μέσον, which we excluded from our discussion of the first figure[21] (cf. p. 97) can now be satisfactorily explained: in the first figure too the middle term is sufficiently determined by its position in the standard formulation: it is the term which "stands in the middle". And it is possible to say this although in the first figure (in contrast to the second and third) the middle term occurs twice: for it appears in the two argument places *in the middle* of the syllogism.

However, it is clear that "major" and "minor" could not have been defined in the first figure, as they are here in the second, by their relative *distance* from the middle term: there they are both *equidistant* from the middle. In spite of this, Aristotle was not compelled to define them by their relative extensions, as he tried to do. He could have defined the major as the predicate, and the minor as the subject, of the middle.

Aristotle's definition of the *third* figure at the beginning of *A* 6 is entirely analogous to that of the second: "(a) If the one belongs to all and the other to none of the same (term), or if both belong to all or to none, I call such a figure the third. (b) I call middle in this figure the term of which both (the other terms) are said (literally: of which both which are said are said); and outer those (terms) which are said (of it), (c) major, the outer which is further from the middle term, minor, the nearer. (d) The middle term is outside the outer terms and is last in position."[22]

When he indicates examples of premiss-combinations which illustrate the formal characteristics of the third figure, Aristotle again shows no interest in the question whether valid syllogisms can be constructed from them. The pairs he names are: *AaB & CeB*, *AeB & CaB*, *AaB & CaB*, *AeB & CeB*. Only the second and third of these yield valid moods (*Felapton* and *Darapti*); the four valid moods of the third figure which contain a particular premiss are not considered. Middle and outer term are defined, as in the second figure, by their function as subject or predicate in the premisses. As in the second figure, major and minor term cannot be distinguished by their functions. Therefore Aristotle again falls back on the order of the terms in a standard formulation: "If both *P* and *R* belong

to all S (then there is a syllogism – the supplement must be taken from 28a15–17; for the expression cf. § 6 p. 18) that the P will necessarily belong to some R" (*A* 6, 28a18–19). S is the middle term, P the major, and R the minor. If we look at this standard formulation we see at a glance what Aristotle means when he says that the major term is *further* from the middle term. And we can see that Aristotle was right to say that here (in contrast to the first figure and contrary to what its name might suggest) the middle term is outside the outer terms and last in position.

These formulations have so little of the esoteric about them that they tend to appear trivial. And it is precisely this apparent triviality that has till now hampered the correct interpretation of Aristotle's definitions.

§ 24. Results of the Analysis. The General Account of the Figures in *Prior Analytics A* 23 and *A* 32

The chief results of our analysis of the text are these: in his discussion of the figures and moods of the syllogism in *APr. A* 4–6, Aristotle defines the terms "middle", "outer", "major", and "minor" *anew* for each figure. In the first figure the relative extensions of the terms form the basis of his definitions. He was probably led to this by noticing certain facts about arguments in *Barbara* with concrete terms and true premisses. We have shown that these definitions are not universally applicable even to syllogisms in *Barbara* and that the terms of the other moods would not satisfy them even if we were willing to confine our attention to syllogisms with concrete terms and true premisses. Indeed, Aristotle's own definition of the term-relation "be contained", shows that a definition by relative extensions is only possible if true judgements of the form AaB are presupposed for all terms. This thought appears to have occurred to Aristotle himself: there is no other explanation of why he replaces the phrase "be contained in" by the undefined "be under" in cases where *particular* premisses occur. In place of the erroneous extensional definitions for the first figure, Aristotle had two choices open to him: (a) he could define all terms by reference to their *function* as subject or predicate in the premisses: the middle term by its property of functioning both as subject and as predicate in the premisses; the outer terms by the negation of this; the major by its appearing only as predicate, the minor by its appearing only as subject; or else (b) he could define the terms by their order in a

standard formulation. Although Aristotle does not do this, he would appear to have made a start in this direction: for in the first figure and there alone he uses *names* ("first" and "last" term) which only make sense on the assumption of the standard formulation for first figure syllogisms.

In the second and third figures Aristotle does use order in a standard formulation to define the expressions "greater term" and "less term" (major and minor). In both figures he first defined "middle" and "outer term" by their different grammatical function as subject or predicate in the premisses. But, since the function of the major and minor terms is here the same, there is no alternative but to define them by their relative ordering in some chosen standard formulation. Once this is given, the middle term too could of course be sufficiently determined by its position in the standard formulation. These definitions are logically impeccable; that is, they are satisfied by all the terms which Aristotle refers to by the defined names.

The fact that in the second and third figures the function of major and minor in the premisses is the same, has led several commentators to infer that Aristotle's definitions pay attention to the *conclusion* of the syllogisms too. It is a fact that in all 14 syllogisms systematically discussed in *A* 4–6, the major term is always the predicate and the minor always the subject of the conclusion. However, as we saw, in his discussion of the figures and in his definitions of the terms in them Aristotle never so much as mentions the conclusion; to illustrate the figures he constructs in each case certain pairs of *premisses*. Aristotle's definitions of "major" and "minor term" *must* therefore be independent of the function of these terms in the conclusion; and that is just what we have established on the basis of our interpretation. Nor does the division of functions in the conclusion *follow* from these definitions. It is rather a mere convention – a convention doubtless grounded on the fact that the 'conventional' order of the outer terms in the perfect first figure syllogism is a necessary condition of their *perfection*. The extension of this convention to the second and third figures is purely arbitrary, and (on Aristotle's assumptions) mistaken: for because of it certain valid syllogisms are treated in *A* 4–6 as invalid. The mistake, however, was corrected by Aristotle himself in the appendix, *A* 7.

The expressions θέσις, τίθεσθαι, ἐγγύτερον, πορρωτέρω, πρός τι κεῖσθαι, etc. which continually reappear in Aristotle's definitions do not

refer to the extensions of the terms, nor to their inner 'relationship', nor to anything of that sort, but to the spatial ordering of the terms in certain standard formulations. Every other interpretation, as we shall show in § 26, would burden Aristotle with logical errors.

Each of these definitions of the terms occurring in a syllogism is valid for only one of the three figures; the definitions are therefore differently constructed in each of the three chapters *A* 4–6. Can we not frame definitions to hold for all three Aristotelian figures? Before examining the texts in which Aristotle presents such general definitions, I shall make some preliminary remarks. First, it is plain that the middle term can be defined validly for all three figures, as the term which occurs in *both* premisses, and the outer terms as those which each occur only in one. However, it is not possible to distinguish the major from the minor term generally for all figures by their function in the premisses. For in the second figure the two outer terms are each subject and in the third they are each predicate of one premiss. We could get round this by taking their grammatical function in the *conclusion* as their distinguishing characteristic: then in all figures the major would be that outer term which occurs as predicate, and the minor that which occurs as subject, of the conclusion. This definition is customary and correct in the framework of the traditional four figures. In Aristotle, however, since he knows only three figures, it would not be universally valid: for he recognises syllogisms of the form $AaB \& BeC \rightarrow CoA$ (*A* 7, 29a23–26) where *A* remains the major term even though it is the *subject* of the conclusion: if it did not, Aristotle could not say that in syllogisms of this form "the minor is said of the major" (γίνεται συλλογισμὸς τοῦ ἐλάττονος ἄκρου πρὸς τὸ μεῖζον: 29a22–23).

It might be objected to this that it is illegitimate to adduce *A* 7 in explanation of *A* 4–6, since it was obviously written after *A* 4–6 and indeed deviates in many other ways from the doctrines expounded there. However, 'unorthodox' syllogisms appear in *A* 5 and *A* 6 too; they are used in the proof of two of the fourteen syllogisms systematically treated, and must therefore have been regarded by Aristotle as valid. The proof of *Camestres* (II) in *A* 5 assumes the validity of the syllogism:

> If the *M* belongs to all *N* and to no *X*, then *X* will belong to no *N*. (27a9–10.)

By the definition of the outer terms of the second figure, X here is the *minor* and N the *major* term: thus the proposed general definition by reference to the conclusion would *contradict* Aristotle's own definition of the second figure. The syllogism used in A 6 to prove *Disamis* (III) is constructed on the same lines:

> If the R belongs to all S and the S belongs to some P, then the R will belong to some P and hence the P to some R. (28b10–11.)

The second syllogism here, $RaS \& SiP \rightarrow PiR$, is (in traditional logic) *Dimatis* (IV) with transposed premisses; in Aristotle's view, however, it is a (supplementary) mood of the *first* figure; and by the definitions for the first figure, R is the *major* and P the *minor* term, although R is here the subject and P the predicate of the conclusion.

A *functional* definition of "major" and "minor term" which will cover all figures is impossible for Aristotle because he only considers the *premisses* in his definitions. The traditional method of distinguishing major and minor term by their function in the conclusion is correct on the supposition of four figures, but in Aristotle it would contradict the special definitions he gives for the individual figures.

Such a general definition could, however, be achieved if we fell back on the *standard formulations* of the syllogisms in the individual figures. The major term would be sufficiently determined as the outer term which in all standard formulations *precedes* the other outer term; the minor could therefore be defined as the outer term which in all standard formulations *follows* the other outer term. Such a definition of "major" and "minor term", which would hold for all figures, was not in fact given by Aristotle. He limits himself to giving a general definition of "middle term" and then characterizing the three figures by reference to this definition. But it is important to notice that corresponding definitions of "major" and "minor term" can be developed without difficulty from Aristotle's assumptions.

I now give the two relevant texts in translation:

1. *A* 32, 47a36–b6: In order to bring a given argument into syllogistic form, "(a) we must first construct the two (propositions which are to play the part of) premisses; then we must analyse them in the way described (each) into (two) terms; we must appoint as middle term the term which

occurs in both the premisses. (b) For it is necessary that the middle term occur in both premisses in all the figures. (c) If the middle term occurs as predicate of one and subject of the other premiss (literally: if the middle term is itself said and something is said of it, or is itself said and something else is denied of it), we shall have the first figure. (d) If it is both said and denied of something else, (we shall have) the middle (i.e. second figure). (e) If other (terms) are said of it, or the one is said of it and the other denied of it, the last (third figure). (f) For this is how the middle term was related in each of the figures."[23] Shortly afterwards he says (b13–14): (g) "We recognise the figure (which is present in each case) by the position of the middle term".[24]

Sentence (b) obviously contains a definition of the middle term valid for *all* figures. And since the outer terms are the terms which are *not* the middle term, we have in the negation of this property a general definition of "outer term". Sentence (c) contains a definition of the first figure which (unlike the one given in *A* 4) is free from all reference to the extension of the terms occurring in its syllogisms, and hence – since, as we saw, the definition proposed in *A* 4 is mistaken – it must count as an improvement.[25]

Sentences (d)–(f) need no commentary. We must note that sentence (g), however, is only true on the assumption that the syllogisms have been put into their standard formulations. To give an example, the syllogism:

> If to all *X* the *N* belongs and the *M* belongs to no *N*, then the *M* will belong to no *X* (*A* 5, 27b18).

would not readily be recognised as *Celarent* (I), just by looking at the 'position' of the middle term. When it is reduced to the standard form:

> If the *N* belongs to every *X* and the *M* belongs to no *N*,

then the first figure can be recognised, in spite of the transposition of the premisses, from the position of the middle term. Similar cases are found at: *A* 5, 27a34–36; *A* 6, 28a20; *APst. B* 16, 98b6.

2. The other passage to be considered is *A* 23, 41a13–18: "If (in order to form a syllogism) it is necessary to find something common to both (outer terms) [i.e. a middle term], and this can be done in *three* ways – either by asserting *A* of *C* and *C* of *B* or *C* of both (*A* and *B*) or both

(A and B) of C – and these are the figures we have described (in A 4–6), then it is clear that every syllogism must be constructed by means of one of these figures."[26]

When Aristotle affirms that only three combinations can be constructed under the given conditions, he is of course wrong. We can easily prove this: from three terms A, B, C, I can first construct four different propositions in which C and one of the other two terms occur. These propositions (rather: propositional schemata) are: AxC, BxC, CxA and CxB (where x is a variable ranging over the relation-constants a, e, i, and o). If I next arrange these four propositions in ordered pairs such that the first member of the pair always contains A and C and the second member always C and B, then I can construct not three but *four* such pairs: 1) AxC, CxB; 2) CxA, CxB; 3) AxC, BxC; 4) CxA, BxC. Aristotle did not take account of the fourth combination when he set out the possibilities. It corresponds, clearly enough, to the traditional fourth figure. Did he, as Łukasiewicz thinks, simply "overlook" it? (AS, p. 23) That would be astonishing – the more so since he allowed the individual moods of the figure to be valid, and appended them as 'supplementary' syllogisms in A 7 and B 1 of the *Prior Analytics*. Is he really supposed to have simply 'overlooked' the fact that a supplementary *figure* offered places for these moods which were standing "homeless" (AS, p. 27) at the door of his system? In the next section I shall point out certain difficulties which (granted Aristotle's assumptions) inevitably militated against the introduction of the fourth figure into his system. In all probability it was these difficulties which made Aristotle retain his three-fold classification; the price of course is that his three figures do not contain all the syllogisms which he admits to be valid – thus confuting the assertion of passage 2 (41a16–18) that all valid syllogisms belong to one of the three figures.[27]

§ 25. Aristotle and the Fourth Figure

As we have already noted in more than one connexion, Aristotle briefly informs us in A 7 that, besides the arguments proved to be valid in A 4–6, the premisses *ie* and *ae* allow the construction of further valid syllogisms in all three figures. The new syllogisms which result from the application of this rule to the first figure, and which Aristotle, according to the wording of his text ("a syllogism is possible *in* all figures"), regarded as

syllogisms *of* the first figure, are explicitly formulated in the text: they are

(1) $AaB \& BeC \rightarrow CoA$ and
(2) $AiB \& BeC \rightarrow CoA$.

Aristotle does not bother to write out the additional moods which accrue "in the second and third figures"; nor does he say that what holds in general for *ie* and *ae* also holds for *oa* and *ao* in the second and third figures. This silence is surprising: the reason for it, we have conjectured (§ 14, p. 55), may be that Aristotle realized that the new moods of the second and third figures transform into other moods of the *same* figure (moods already discussed in *A* 4–6) if their conclusions are 'disconverted' by interchanging *A* and *C* in premisses and conclusion. Thus the new mood of the second figure, $BiA \& BeC \rightarrow CoA$, changes to $BiC \& BeA \rightarrow AoC$, and this, after transposition of the premisses (since the conventional order begins with the premiss containing *A*), becomes $BeA \& BiC \rightarrow AoC$ – which is the standard form of *Festino*. Similarly, the additional mood of the third figure, $AaB \& CoB \rightarrow CoA$, can be turned into *Bocardo* (III) by interchanging *A* and *C*. This operation is, of course, just the one which the law:

$$R|S \subseteq T \leftrightarrow \breve{S}|\breve{R} \subseteq \breve{T} \quad \text{(cf. § 14, pp. 55–56),}$$

makes possible for products of all dyadic relations. Application of it to the two additional moods (1) and (2) of the first figure does not produce moods of the same figure already treated in *A* 4, but rather (1′): $BeA \& CaB \rightarrow AoC$ and (2′): $BeA \& CiB \rightarrow AoC$, which do not coincide with any of the first figure moods. If, as is commonly the case, Aristotle's recognition of (1) and (2) is taken as an admission of the moods *Fesapo* and *Fresison* of the traditional fourth figure (Łukasiewicz, AS, p. 26; Ross, APPA, p. 314; Bocheński, FL, p. 82; HFL, p. 70; cf. also Überweg, SdL[5], p. 338), then it is clearly assumed that Aristotle saw the equivalence of (1) with (1′) and of (2) with (2′), and therefore that he also saw the possibility of transforming 'indirect' moods – moods with converse conclusions – into 'direct' ones by means of the operation we have described.

In *A* 7 Aristotle explicitly formulates syllogisms supplementary to those expounded in *A* 4–6. In *B* 1, 53a3–14[28], he does not do this, but instead remarks (correctly) that a syllogism, the conclusion of which logically entails its *converse*, can also have this converse proposition as its conclusion. The 'Rules of Conversion' which Aristotle briefly repeats in this

passage are based on the logical implications between propositions of the form AxB and BxA (where x ranges over a, e, and i), which he has discussed in detail in *APr. A* 2 (cf. below, § 28, p. 138). AeB entails BeA; AiB entails BiA (the traditional 'pure' conversions); AaB entails BiA ('impure' conversion); AoB entails nothing: "for it is ('relatively') possible that B belongs to all A" (53a14). It is further evident that, if two propositions together entail a third, they also entail any proposition which the third entails. This is a law of *propositional* logic:

$$[(pq \rightarrow r) \& (r \rightarrow s)] \rightarrow (pq \rightarrow s).$$

This can be derived, by substitution of "$p \& q$" (abbreviated to "pq") for "p", "r" for "q", and "s" for "r", from the law:

$$[(p \rightarrow q) \& (q \rightarrow r)] \rightarrow (p \rightarrow r),$$

which is the so-called 'hypothetical syllogism' (Whitehead and Russell, PM I, p. 112, prop. 3.33). We shall consider Aristotle's use of laws of *propositional logic* in detail in chapter V; it is enough here to state that, according to Aristotle's text, three new moods *of the first figure* are gained by converting the conclusions of *Barbara*, *Celarent*, and *Darii*:

(3) $AaB \& BaC \rightarrow CiA$;
(4) $AeB \& BaC \rightarrow CeA$; and
(5) $AaB \& BiC \rightarrow CiA$.

In the other two Aristotelian figures additional moods of course result – in all those cases where the conclusion of a syllogism has the form AeC or AiC (AaC occurs only in the first figure).

We would expect Aristotle to point out here that still further moods can be derived from all syllogisms with a universal conclusion, AeC or AaC, by virtue of the 'Laws of Subalternation' ($AaB \rightarrow AiB$ and $AeB \rightarrow AoB$). These would be the five traditional 'subaltern' moods, *Barbari*, *Celaront*, *Cesaro*, *Camestrop* and *Celantop*.[29] Aristotle does not mention this possibility. We may note in this connexion that, although the *Analytics* deals at length with the rules of conversion, it does not do so with the laws of subalternation. However, even in the *Analytics* Aristotle *uses* the proposition $AeB \rightarrow AoB$: ἐπεὶ γὰρ ἀληθεύεται τὸ τινὶ μὴ ὑπάρχειν τὸ M τῷ Ξ καὶ εἰ μηδενὶ ὑπάρχει ... (A 5, 27b21–22); and $AaB \rightarrow AiB$ can be deduced from the laws of conversion: from $AaB \rightarrow BiA$ and $BiA \rightarrow AiB$ it follows by the logical rule stated above (the hypothetical syllogism)

that $AaB \rightarrow AiB$. Both propositions are presupposed in Aristotle's procedure for proving the invalidity or inconcludence of certain pairs of premisses (cf. § 31, p. 176).

The recognition of (3), (4) and (5) has again been taken by most commentators as an admission of the fourth figure moods *Bamalip*, *Calemes* and *Dimatis* (Łukasiewicz, AS, pp. 25–26, Bocheński, FL, p. 83; HFL, p. 71; Ross, APPA, p. 314, Überweg, SdL^5, p. 339); this view is, of course, reasonable only on the assumption that Aristotle realised how these syllogisms can, by interchange of A and C, be put into normal form (with unconverted conclusions).

Aristotle treated moods (1) to (5) as valid; in two cases ((1) and (2)) he offered explicit formulations, for the rest he gave and explained the principles on which they could be constructed. Their premisses have (in Aristotle's formulation) the form of the first figure; but their *conclusions* violate the tacit convention of *A* 4–6 by which the *major* term must be the predicate of the conclusion. This discrepancy could be avoided by changing (1) and (2) into the equivalent (1′) and (2′) and by bringing (3)–(5) into corresponding normal forms (3′)–(5′). In these syllogisms the major term *would* be predicate of the conclusion, as Aristotle requires in *A* 4–6. However, their *premisses* would then no longer answer to the definition of the first figure given in *A* 4. For the last term would not be contained in the middle term, but vice versa.[30] And if we replaced the erroneous definition in terms of extension by the later definition of *A* 32, then *Fesapo* and *Fresison* – (1′) and (2′) – for the reasons given on pp. 129–130 n. 25, would not be covered.

It is scarcely probable, in my opinion, that Aristotle failed to see the shadow cast on his syllogistic system by the fact that certain of the arguments he expressly recognises as valid resist distribution into the three figures he discusses in *A* 4–6, despite his explicit affirmation in *A* 23 that each valid syllogism belongs to one of these three figures. The texts make it clear that *despite his definitions* Aristotle treated (1)–(5) as moods of the *first* figure. And as a matter of fact, if one of the figures had to accommodate these moods, the first alone would be a possible host. For only in this figure, on Aristotle's view, does the middle term occur once as subject and once as predicate of a premiss. If the definition of the first figure is reduced to this one condition, then the inclusion of moods (1)–(5) raises no difficulties. Theophrastus followed this path consistently

and in his systematic exposition of syllogistic he added the arguments in question as moods 5 to 9 of the first figure in the order (3), (4), (5), (1), (2). He expressly abandoned Aristotle's assertions both about the extensions of the terms in the first figure, and also on the perfection of all (assertoric) first figure moods; he rejected the condition stated by Aristotle in *A* 23 that in the first figure the major term must be said of the middle and the middle of the minor. He restricted himself to the statement that the first figure is present when the middle term is predicate in one and subject in the other premiss (cf. Alex. *in APr.* 258, 17–25; on which, Łukasiewicz, AS, p. 27).

There is no doubt that Theophrastus' operation removes the difficulties we have noticed and that it does so with the least possible surgery to Aristotle's system. A logician of Łukasiewicz's stature could write: "The correction of Theophrastus is as good a solution of the problem of the syllogistic figures as the addition of a new figure" (AS, p. 28).[31] However, because of a certain asymmetry in the resulting system this 'correction' seems to me to be bought at too high a price. Sauce for the goose is sauce for the gander: if syllogisms with converse conclusions are allowed in the first figure, why may not all syllogisms formed by converting the conclusions of other arguments claim a place in the system? The trick transformation of the fourth figure moods into 'indirect' or 'supplementary' moods of the first figure is meretricious: it charms to deceive. The fourth figure does not disappear: just as the direct moods of the fourth figure can be presented as indirect moods of the first, so the direct moods of the first may be construed as indirect of 'supplementary' moods of the fourth. The possibility of counting the direct moods of the fourth as indirect moods of the first does not establish the priority of the first figure nor the superfluity of the fourth. It is possible, of course, on certain assumptions, to make do with three figures and still bring all valid moods into them: we need only make the following *stipulative* definition: a syllogism which can be derived from a valid syllogism with the help of the law of relational logic: $R|S \subset T \leftrightarrow \tilde{S}|\tilde{R} \subset \tilde{T}$ shall be treated as identical with that syllogism. Then we need either the set of indirect moods or the set of direct moods of both the second and the third figures (the omitted set of each figure can be derived from the included set by application of the law); in addition we need only either all the moods of the first figure or all the moods of the fourth (since the moods of the omitted figure can be derived from the included figure).

We have established the following points: 1. Aristotle recognised, in addition to the moods treated in *A* 4–6, all the syllogisms of the traditional fourth figure – or rather their indirect equivalents. 2. He most probably realized that indirect syllogisms can be changed into direct ones by interchanging *C* and *A* in their conclusion and premisses. 3. In *A* 23 he gives a formal description of the possible figures in such a way that the *omission* of one possibility is immediately apparent: this is the traditional fourth figure. 4. He implies that these additional moods belong to the first figure although they do not fit his definitions of first figure syllogisms.

These facts taken together seem to me to exclude the possibility that Aristotle simply overlooked the fourth figure, or that, after his discovery of the fourth figure syllogisms, he did not have the time[32] to accommodate them in his system by the adjunction of a new figure. It is more reasonable to suppose that he had certain objections to the introduction of a new figure. To this extent the oft-repeated assertion that Aristotle *rejected* the fourth figure is not entirely false. But the reasons for his rejection are fundamentally different from those which have hitherto been given. It has been opined that the fourth figure is logically invalid,[33] or 'unnatural',[34] or epistemologically worthless.[35] The first view is logically false, the second and third logically irrelevant. Rather, Aristotle did not admit the fourth figure into his system because he was unable to *define* it by the methods he had developed. This must now be proved.

Let us suppose that Aristotle had wanted to insert a chapter between *A* 6 and *A* 7, introducing the fourth figure in the same way as he had introduced the first, second, and third figures.

Perhaps he would have begun on the pattern of *A* 4, 25b32–5 (cf. p. 91), and tried (incorrectly) to define the figure by the relative extensions of its terms: "When three terms are so related to one another that the middle is or is not in the last as in a whole and the first is or is not in the middle as in a whole, then there is necessarily a syllogism (not a perfect one) of the outer terms." Here again the same mild paradox troubles us: the 'last' term in this formulation occurs *before* the 'first'. But here the paradox cannot be cleared up by looking to a formulation using the constant "be said of", or to a standard formulation. For if we reformulate our statement by means of the definition of "be contained in" (cf. p. 92), we get: "When three terms are so related to one another that the middle is

said of all or none of the first and the last is said of all or none of the middle, then there is a syllogism of the first term with regard to the last". And the standard formulation (in which, following Aristotle's practice of using different variables for the different figures, the three terms are expressed by *I*, *K* and *L*) must read thus:

> If the *I* is said of all *K* and the *L* is said of all *I*, then the *K* must belong to some *L*.

In this formulation the two outer terms are palpably not in first and last but in second and third positions. First and last positions are occupied by the *middle* term – the 'first' and the 'last' terms of the standard formulation are one and the same.[36]

Aristotle would have eschewed a definition of the three terms of the fourth figure by means of their relative *extensions* just as he did in the second and third figures – but for different reasons. As we have seen, he was encouraged to this enterprise in the first figure by the fact that the terms of concrete syllogisms in *Barbara* with true premisses do as a matter of fact fulfill the extensional definitions which he drew up for the *whole* first figure. Since in the second and third figures no such determinate relations of extension hold for any mood, Aristotle no longer attempted to give extensional definitions. What holds for *Barbara* holds also for *Bamalip* (IV) with concrete terms and true premisses: however, an analogous definition of the terms of the fourth figure would have been verbally identical with that of the terms of the first (*A* 4, 25b35–37).[37] This sort of definition, quite apart from its logical inadequacy (cf. § 22, p. 98), is ruled out: it does not allow Aristotle to distinguish the fourth figure from the first.

We are left with a definition by means of the grammatical *function* of the terms in the premisses or one based on the order of the terms in a *standard formulation*. Let us try the first possibility. Aristotle used it to define the *first* figure in *A* 32, 47a40–b1, the passage we have discussed (though he did not do so in *A* 4): we have already established that the definition proposed there covers the traditional fourth figure too (cf. p. 129, n. 25). Here again Aristotle would have had either to give verbally identical definitions for both the first and the fourth figures, or else to emend his text in such a way that the definition of the first figure did not coincide with that of the fourth.

Aristotle must therefore have supposed that the fourth figure definitions could only be based on the *order* of the terms in the premisses. He had already followed this path for the second and third figures – clearly because in these figures the function of major and minor terms in the premisses is identical, while the conclusion is not, in his system, germane to the determination of a figure or the definition of the terms of its syllogisms.

However, this path required recourse to an *artifice*: whereas in the traditional formulation of the syllogism the middle term always occurs *twice* – thus: "*A* belongs to all *B*; *C* belongs to all *B*" and "*B* belongs to no *A*; *B* belongs to all *C*" – so that the middle term in the one case occupies second and fourth, in the other first and third position, Aristotle chose a standard formulation of the premisses in which the middle term, like each of the outer terms, occurs only *once*. Aristotle's standard form of the second figure is: "If the *M* belongs to no *N* and to all *X*"; and of the third: "If the *P* and the *R* belong to all *S*". Only so is he able to say that the middle is in the second figure *the* first term and in the third *the* last term, and to *define* it by these positions. Only so can he distinguish the outer terms, which have the same function in the premisses, by their greater proximity to or distance from the middle term. In the first figure too (in a clause which was probably introduced into *A* 4 after he had discussed the second and third figures) Aristotle defines the middle term as that which "stands in the middle". For, although the standard formulation of the first figure must contain two occurrences of the middle term because of the change in its function, nevertheless the two places in which it occurs are immediate *neighbours*. If this were not so the assertion that the middle term "stands in the middle" would lead to difficulties.

Any standard formulation of the fourth figure, as of the first, must introduce the middle term twice, because of its different grammatical function in the two premisses. In this case, however, it is *impossible* to juxtapose its two occurrences (cf. p. 131, n. 37). And hence Aristotle could not here give an unequivocal definition of the middle term by means of its position in the standard formulation. Still less could he distinguish the major and minor terms in this way. It might at first be thought that the major term, which in the *first* figure is the first term in the standard formulation, could be the *last* term in the fourth; but a glance at the standard formulation –

If I belongs to all K and L belongs to all I ... –

will show that here the *middle* term occupies last place (and first place too). The same holds for the minor term. It is impossible to distinguish major and minor terms by their greater or less distance from the middle term (as is done in figures II and III) since 'the' place of the middle term cannot be uniquely determined, and major and minor are each nearer to one and further from the other occurrence of the middle.

Thus it was quite impossible for Aristotle to define an additional fourth figure in a way *analogous* to the definitions of *A* 4–6. Definitions by means of the *extension* of the terms or of their *function* in the premisses would have coincided with the definitions given for the first figure; and an attempt based on the order of the terms in a *standard formulation* must have foundered on the impossibility of giving a unique specification of 'the' position of the middle term in the premisses. These considerations (and, I think, these alone) explain why Aristotle "did not recognise a fourth figure". An additional figure simply does not fit into the polished system of *A* 4–6.

However, certain slight modifications to the definitions given in *A* 4–6 and *A* 32 would have made it easy for him to set up a system containing four figures; and to do this he need not have followed the path proposed by Philoponus and accepted as the only solution by Łukasiewicz – definition of the major and minor terms by their function in the *conclusion*. Aristotle has already given a definition of the middle term valid for *all* figures ("the term that occurs in both premisses", *A* 32, 47a38–39): he need only have added the stipulation that the major (minor) term is that outer term ("outer" = "non-middle") which in the standard formulation *precedes* (*follows*) *the other outer term.* He could then have defined the first figure as that in which the middle term is subject of the major and predicate of the minor; the second as that in which the middle term occurs only as predicate; the third as that in which the middle term occurs twice as subject – and the fourth could then have been readily defined as that in which the middle term is predicate of the major and subject of the minor. We have already seen that Aristotle did not take this path. In *A* 32, 47a38, he did not even offer our proposed definitions of major and minor terms which would have been valid for all figures alike. He started out from the individual figures and tried to *describe* them or their indi-

vidual syllogisms, not to derive them all from one principle. He first investigated the syllogisms of the first figure and inferred, wrongly, that certain extensional relationships which do always hold in the case of some first figure syllogisms, must characterise all the several figures. When he came to the second and third figures, this belief must have been shaken, if only by the fact that, since the extensional relationships holding between given terms always remain *constant*, when syllogisms of the *second* and *third* figures are transformed into syllogisms of the *first* figure, this transformation cannot alter the extensions of their terms. At all events, in *A* 5 and *A* 6 Aristotle turned to giving heterogeneous definitions based on the grammatical function of the terms and their spatial order in a standard formulation. The fact that second and third figure syllogisms can be formulated in such a way that the middle term occurs only once, was a signal aid. And the fact that this is not possible in the case of the fourth figure was a weighty argument against the expansion of his system by the addition of a fourth figure, the syllogisms of which he had already recognised in *A* 7 and *B* 1. The fact that the definitions for the fourth figure must in part have overlapped those given in *A* 4 and *A* 32 for the *first* may have confirmed him in the belief that at least no important logical distinction was blurred by treating the new moods as members of the first figure.

This would give us a new answer to the old question: why did Aristotle not admit a fourth figure into his system?

§ 26. Survey of Alternative Solutions to the Problem

All previous interpretations differ from the present one in that they do not bring out the importance of the *standard formulations* for Aristotle's definitions of σχῆμα, μέσον, ἄκρον, μεῖζον and ἔλαττον. Hence, the expressions θέσις, ἐγγύτερον, πορρωτέρω, etc., which are crucial to the understanding of Aristotle's theory of the figures and thus of his syllogistic as a whole, had either to be disregarded completely, or to be construed as *metaphorical* expressions for extensional or functional relationships between the terms, or else, if their original *spatial* meaning was to be retained, to be referred to certain, entirely conjectural, *models* by means of which Aristotle, like Leibniz, Lambert, Euler and Venn in later ages, was supposed to have illustrated and illuminated the syllogistic moods.

The ancient commentators discuss all this, principally in connexion with the question how in the second and third figures the outer terms, which have the same function in the two premisses, can be distinguished from one another. They asked whether the second and third figures possess major and minor terms θέσει or φύσει (Philoponus, *in APr.* 87, 17).[38] We may suppose that the mere presence of the word "θέσις" in Aristotle's definition of the middle term (*A* 4, 25b36) reminded the commentators of the opposition between θέσις and φύσις which (particularly in the explanation of the origins of language) played a prominent part in the debate between the Stoics and the Epicureans.[39] A major term is such φύσει, in Philoponus' eyes, if it can be defined by its *function* in the premisses, not by the size of its extension. Before him, Alexander in his explication of the definitions of μεῖζον and ἔλαττον at *APr. A* 4, 26a21 (*in APr.* 60, 12 sq.) had spoken, not of the *extension*, but of the *function* of these terms as subject or predicate of the premisses. Alexander clearly realized that definition of these terms by their comparative extensions raises problems – and that these arise even in the first figure; his long discussion of extension (*in APr.* 47, 27–50, 22) leads him to the conclusion that the "greater term" is that which "does not necessarily have a smaller extension" and the "smaller term" that which "does not necessarily have a greater extension" (47, 29–48, 6 and 49, 27–50, 8).[40] His practice of ignoring definitions by extension in favour of definitions by function is followed by Philoponus (73, 3–5 ad *A* 4, 25b35 and 78, 3–5 ad *A* 4, 26a21) – although Philoponus does not state reasons for his choice.

Herminus, Alexander's teacher, tried to find a 'natural' major term for the second figure too. His attempt is reported and criticised by Alexander (in one sentence forty lines long, 72, 26–74, 6). Herminus clearly wanted to take the words "ἐγγύτερον" and "πορρωτέρω" in our text to apply, not to the *spatial* distance of the two outer terms from the *middle* term, but to their *systematic* distance from a *superordinate* term common to the two outer terms. Since it is not always easy to find such a term common to the two outer terms, and since the criterion is useless if the outer terms are different *species* of a genus, Alexander rightly rejected Herminus' theory. However, Alexander's own solution of the problem cannot satisfy us either. According to him, the major term cannot simply be defined as the predicate of the conclusion as, on his explicit testimony (75, 10 sqq.), some of his contemporaries thought. Alexander stands out

against the consequences of this interpretation: for example conversion of the conclusion of *Cesare* (II) *interchanges* the major and minor terms – the original major becomes the minor and vice versa. He prefers instead to take every syllogism as the proof of a *given* proposition: the subject of this proposition is the minor term and the predicate the major – and this remains so even if the conclusion is later converted. Łukasiewicz (AS, p. 31–2) pointed out the arbitrary nature of this stipulation: often the task is not to prove a given conclusion by appropriate premisses but the reverse, to *deduce* a conclusion from given premisses. Furthermore, Alexander's proposal is thoroughly unaristotelian: we have seen (pp. 101 and 105) that Aristotle never takes the conclusion into consideration when he defines the terms in question in *A* 4–6.

Philoponus too (*in APr.* 87, 2-19) offers a suggestion which would explain without reference to a standard formulation why Aristotle asserts that in the second figure the major term is "next to" the middle while in the third the minor is "nearer" to it. Since Herminus' attempt to refer these expressions to the order of the terms in a conceptual pyramid had been refuted by Alexander, Philoponus refers the 'proximity' and 'distance' from the middle term to a certain functional relationship between the middle term and the outer term 'nearer' to it: in the second figure the major is 'nearer' the middle because the middle term is predicate in both premisses and the major is predicate once at least, namely, in the conclusion. Similarly, in the third figure the minor term is 'nearer' the middle, which is subject of both premisses, because it is subject once at least (in the conclusion) whereas the major functions only as predicate (in the first premiss and in the conclusion). This ingenious conjecture suffers from two serious deficiencies: first, Aristotle cannot possibly have assumed that a reader would understand what he meant without being offered some explanation of the metaphorical character of "near" and "far"; secondly, we must again object that every interpretation of Aristotle's definitions must, if it is not to misconstrue them, be independent of any reference to the conclusion, which Aristotle never considers in *A* 4–6. The same objection holds against the definition of major and minor terms by means of their function in the conclusion, which Philoponus offers (67, 14–30) as valid for all three figures and which Łukasiewicz calls "classical" (AS, p. 32). Of course, the definitions proposed by Philoponus for the three figures and the major and minor terms in them is

logically faultless. But it is not an *interpretation* of Aristotle's definitions: it is inconsistent with the way in which in *A* 7 Aristotle introduces the 'indirect' moods as moods of the first figure and says that in their conclusion "the minor term is said of the major"; and it is contradicted by the fact we have so frequently mentioned that the conclusion of the syllogism is irrelevant to the definitions of *A* 4–6.

However, once it was realised that *extensional* definitions do not fulfill their function, since they are subject to obvious objections, and that the *grammatical* function of the terms in the premisses does not allow the outer terms in the second and third figures to be distinguished from one another, it could well be imagined that the solution proposed – or rather revived[41] – by Philoponus was the only one possible.

The accounts of Trendelenburg and Maier, which agree in the main, were given in outline at the beginning of the chapter. Both find Aristotle's extensional definitions for the first figure correct, and both go further than Aristotle – while still purporting to interpret his text – in setting the same erroneous notion which they both embraced at the root of his definitions of the *second* and *third* figures and of the terms appearing in them. In these cases, as we have seen, Aristotle essayed a different type of definition; and he did this because he saw that the type which was possible in the first figure could not be transferred to the second and third. Trendelenburg, in attacking the current interpretation (that of Philoponus) was able to appeal to the correct and important point that Aristotle did not make use of the conclusion in framing his definitions. However, following the fashion of the time, he expressed this fact as a universally valid rule and maintained that Philoponus' procedure was illegitimate as well as unaristotelian: "This arrangement ... perverts the natural relationships, since the conclusion, following from the premisses, cannot possibly turn round and affect its own grounds (the premisses)" (*Log. Unt.* II^3, p. 344). Again, since Trendelenburg did not grasp the role of the standard formulation in Aristotle's definitions, his rejection of Philoponus' explanation could only lead him back again to the extensional definitions of the first figure and therefore force him to suppose that Aristotle "held fast to the inner principle of the subordination of the terms" in the second and third figures too (*Log. Unt.* II^3, p. 343). He himself conceded that supposing the middle term in the second figure to be the first because it is the 'highest' term is "rather the supposition

of an analogy than strictly true" (ib. p. 347) – that is, in more sober terms, it is a false analogy.

The same thesis was defended by Maier against the objections of Überweg; his defence was conducted with great emphasis and admirable prolixity. In particular, Maier referred the expression θέσις, which we saw denotes the *position* of the middle term in the *standard formulation*, to the order of the terms in a conceptual pyramid of increasing generality: "Since the thesis always orders the syllogistic terms in a series, a system of subordination must underlie each of the three figures, in which the second term (*B*, *N*, or *P*) is thought of as subordinate to the first (*A*, *M* or *Π*) and the third (*Γ*, *Ξ*, or *Σ*) to the second" (SdA II, 1, p. 60; similar expressions at pp. 56, 71). "Hence in all systems the major term is that which is higher, more general, than the minor" (ibid.). To the obvious objection that Trendelenburg noticed and that we have already met in Alexander (that nothing can be said of the relative extensions of the terms in a *negative* proposition), Maier replied that "negative syllogistic propositions too have at least the external form of subordination" (II, 2, p. 60, n. 1). And that, he says elsewhere, "is enough for the grasping of the principle" (p. 50). In the same sense a negative proposition would have "at least the external form of affirmation". This line of argument, which not only takes as correct Aristotle's erroneous definitions of the first figure, but also forcibly grafts their erroneous presuppositions on to his correct definitions of the other figures, can stand neither as an interpretation of Aristotle nor as a syllogistic theory in its own right. The logical curiosities among which Maier strays in his detailed discussion of this theory have been sufficiently dealt with by Łukasiewicz (AS, pp. 36–38); I may therefore be excused further catalogue. The methodological point which Überweg made against Trendelenburg is appropriate here too: "This position is mistaken and mistakes may only be imputed to Aristotle's theory if his words admit no other natural meaning, and then only to the extent to which his words compel us" (SdL^5, p. 334).

Überweg therefore decided that Aristotle's criterion for dividing the terms into middle, major, and minor was their function as subject or predicate both in the premisses and also in the conclusion (since the major and minor cannot be distinguished in the second and third figures by their function in the premisses). For, he argued, if we are unwilling to explain Aristotle's definitions of major and minor terms in the two last

figures by reintroducing false assumptions about their relative extensions, then we are left to suppose that "he allowed his definitions (in the main unconsciously) to refer to the form of the conclusion, which later logicians explicitly took as the ground for the distinction between terminus major and minor and hence between major and minor premiss, even though Aristotle does not admit the reference in his division of the figures" (SdL[5], p. 336).

However, we have already seen that in Aristotle's view the definitions of the major and minor terms are *independent* of their function in the conclusion, and that his definitions do not even predetermine the role these terms play in the conclusion. However, since it was not realized that Aristotle's convention of regarding the major term as 'the' predicate of the conclusion owed its origin to the formal properties of perfect syllogisms as such, it was inevitably thought that this convention was connected with the *definitions* of the major and minor terms. It was then only natural to suppose that the difficulties which these definitions occasioned could be overcome by the assumption we have sketched. Furthermore, those interpreters who saw the question of the 'principle' of Aristotle's definitions as an alternative between conceptual hierarchies and grammatical functions, were bound to take an argument against the one as an argument in favour of the other. But there were arguments against both alternatives – so that it seemed necessary to defend Aristotle from himself, either by extending the hierarchical principle to the second and third figures, or else by assuming an 'unconscious influence' from the form of the conclusion. The interpretation proposed in §§ 22–24 which finds the 'principle' in the arrangement of the terms in fixed and designated standard formulations, cannot boast that it synthesizes these antitheses and thus destroys them; but it does at least unshackle the interpreter by proving that there is a third horn to the dilemma.

The interpretation offered by Ross (APPA, pp. 301 sqq.) follows in the main, as we have said, that of Maier. Ross holds – and in this he is indebted to Einarson[42] – that Aristotle's use of the word σχῆμα to signify the figures is an indication that he illustrated his theory of the syllogism by means of *diagrams* which were modelled on the diagrams customarily used by the Greek geometricians in the theory of proportions. Julius Pacius, in his commentary on *A* 4, 26b33, had already conjectured that such diagrams once formed a part of Aristotle's text.[43] According to

Ross, or rather Einarson, Aristotle symbolised the three terms of a syllogism by parallel lines of different lengths, so disposed that the longest was on top, the shortest at the bottom and the middle in the middle. This picture will explain all Aristotle's specifications of the figures and the terms occurring in them. In the first figure the major term is represented by the top, that is the longest, line, the middle term by the middle line and the minor by the bottom and shortest of the three. In the second figure the major and middle, in the third the middle and minor change places with respect to their diagrammatic representations. The relative length of the lines thus always corresponds to the relative extension of the terms. All this presupposes, as Ross himself admits (APPA, p. 301), that even in the case of negative and particular propositions the predicate is treated as having a *greater extension* than the subject; and that it is so treated "by analogy" with true propositions of the form *AaB* in which the predicate is in fact more extended than, or at least co-extensive with, the subject. "Thus any term which in any of the three propositions appears as predicate is treated as being more general than the term of which it is predicated" (APPA, p. 302). Thus Ross too extends the mistaken extensional definitions of the first figure to all three figures; and he does worse: contrary to Aristotle's practice he adduces the *conclusion* in his definitions. The expressions ἐγγύτερον, πορρωτέρω, πρὸς τῷ μέσῳ κείμενον, ἔξω τῶν ἄκρων, etc. and the determination of the middle term by its θέσις are referred by Ross to his conjectural diagram: expressions which Aristotle introduced precisely to avoid the logical errors of extensional definitions are explained and founded on these very definitions (APPA, p. 307).

Ross' interpretation does have one important advantage over Maier's: it expressly retains the *spatial* meanings of the words in question and eschews any *metaphorical* reference to an "affinity of nature". In fact there must always be a σχῆμα in which the terms are arranged if the spatial expressions are to bear any meaning at all. But there is no reason why this σχῆμα should be a particular geometrical *diagram* of the syllogistic 'figures'. No doubt it would be wrong to ignore the evidence of Philoponus (*in APr.* 66, 27–67, 13) who asserts that Aristotle, as φιλογεωμέτρης, did his best to make his theory of the syllogism as like as possible to a geometrical system, and who goes into the parallels between ὅρος and στιγμή, διάστημα and γραμμή. (Ross, by the way, does not

cite him.) On the other hand, σχῆμα need not always mean in Aristotle a geometrical figure. Bonitz' *Index* (739b30–740a42) gives sufficient proof of this. According to Bonitz σχῆμα means in the first instance a spatial shape, but then "omnino formam formaeque rationem et varietatem" (739b42). Thus Aristotle speaks of a σχῆμα τι δημοκρατίας (*Pol. Z* 4, 1318b26), that is "a *kind* of democracy"; again, closer to our text, he says "τὸ σχῆμα τῆς λέξεως δεῖ μήτε ἔμμετρον μήτε ἄρρυθμον εἶναι" (*Rhet. Γ* 8, 1408b21), where he means the *form* of a piece of prose. The σχῆμα of a syllogism, then, is, according to all the evidence we have for the use of the word, *not* a geometrical figure drawn to illustrate it, but *its own external form*: otherwise Aristotle could hardly say that a proof is given "by means of the first *figure*", when he means that it is given by means of a *syllogism* in the first figure, that is, by means of a syllogism which *has* the form which Aristotle *calls* the first figure. Waitz, it may be noted, had already shown that it is unnecessary to take the word σχῆμα to refer to a geometrical diagram; he related it to the order of the terms in the syllogism: "Equidem hanc vocem non tam a geometris petitam quam de ipso ordine terminorum accipiendam putaverim" (I, p. 384).

It is surprising that Ross, immediately after giving the interpretation we have discussed, continues: "It may be added that in A's ordinary formulation of a second-figure argument ... the major term *N* is named next after the middle term *M*, while in the ordinary formulation of the third figure ... the minor term *P* is named next before the middle term *Σ*" (APPA, p. 307). On the same page the same supplementary explanation ("in his ordinary formulation") follows the interpretation claimed as uniquely correct ("in his diagram") in the discussion of the sentence: τίθεται δὲ τὸ μέσον ἔξω τῶν ἄκρων, πρῶτον δὲ τῇ θέσει (*A* 5, 26b39). It might be conjectured that, had Ross said "standard formulation" instead of "ordinary formulation", he would have turned his second interpretation into a "standard interpretation", and gladly avoided the hypothesis of a diagram, which had in any case to be based on a mere analogy, and which is not necessary to the understanding of the text.

We have already noted that Łukasiewicz' treatment of the problems of this section is essentially the same as that of Überweg (whom Łukasiewicz does not name). Łukasiewicz' criticism of Maier's theory had, in all but a few details, been anticipated in Überweg's confrontation with Trendelenburg; and Überweg's thesis that Aristotle distinguished major

from minor term (albeit "in the main unconsciously") by their function in the *conclusion* recurs almost verbatim in Łukasiewicz: "Aristotle does not give a definition of the major and minor terms valid for all figures; but practically he treats the predicate of the conclusion as the major term and the subject of the conclusion as the minor term" (AS, pp. 29–30). This sentence contains a kernel of truth – but the truth is obliquely expressed. Aristotle does indeed "practically" treat the major term as the predicate and the minor as the subject of the conclusion. But this convention has nothing to do with his *definitions* of these terms: if he gives no definition of them which is valid for all figures, he still gave perfectly satisfactory and correct definitions – relative to certain standard formulations – for each figure individually. We have already shown that Aristotle employed the conclusion in his definitions neither theoretically nor "practically", neither consciously nor "unconsciously".

Łukasiewicz' discussion is in many ways superior to those of his predecessors (with the exception of Lambert and Überweg); above all he realized clearly and stated uncompromisingly that the so-called fourth figure is just as valid as the first and that Aristotle did recognise its *syllogisms* to be valid. Hence the usual explanation of why Aristotle did not admit a fourth figure was not acceptable: it could no longer be seen as a sign of peculiar wisdom. Therefore it had to be, Łukasiewicz thought, a simple "oversight", "a mistake" (AS, pp. 23; 28). Against this I have tried to show that Aristotle's presuppositions opposed, and their retention rendered impossible, the introduction of the fourth figure.

In sum: in *A* 4–6 Aristotle defines the figures and the terms appearing in them in different ways. In the first figure he mistakenly draws on the relative *extensions* of the terms; in the second and third he turns to the grammatical *function* of the terms in the premisses and their arrangement in a determinate formulation of the syllogisms – what I have called the standard formulation. Since the *perfection* of a syllogism requires that the major term function as predicate in the conclusion, Aristotle generalised this demand to cover all syllogisms; and because of this false generalisation the systematic discussion of *A* 4–6 excludes some syllogisms as invalid which in fact are valid. However, Aristotle appended these syllogisms in *A* 7 and *B* 1, some explicitly and the others by stating the principles by which they can be derived from arguments already known. Most of these new syllogisms can be reduced by conversion (or by the inter-

change of *A* and *C* in premisses and conclusion) to other syllogisms already discussed in *A* 4–6. This does not hold, however, for the syllogisms which can be derived from the *first* figure by means of conversion and the rules described in *A* 7: after conversion they form a new figure – the traditional fourth figure. Such a figure, however, could not be defined without profound changes in the definitions already given in *A* 4–6. For this reason, we concluded, Aristotle treated the new moods – not explicitly but implicitly – as moods of the *first* figure, although their terms do not satisfy the definitions he had given in *A* 4. Theophrastus followed his master in this, expressly adding the five new moods to the first figure. This move, praised by Łukasiewicz, introduces an asymmetry into the system it creates, which then contains the 'indirect' moods of the *first* figure but not those of the *second* or the *third*. The lack of a fourth figure in Aristotle is thus not due to negligence; still less does it result from a belief that the fourth figure moods are invalid, or 'scientifically worthless': the fourth figure is not found in Aristotle because it cannot be defined within the framework of the system he develops in *A* 4–6.

NOTES

1. "Aristotle has overlooked this possibility" (AS, p. 23); "His only mistake is the omission of these moods in the systematic division of the syllogisms" (AS, p. 27).
2. I.M. Bocheński, *La Logique de Théophraste*, 1947, p. 59: "It seems very likely, on the other hand, that *APr. A* 7 and particularly *APr. B* 1 were composed at some time later than the composition of *APr. A* 4–6. Aristotle would no longer have had the time to work out systematically the new discoveries which he had briefly indicated." Why is Aristotle supposed not to have had the time? It is true that, although the *Analytics* are cited by the *Metaphysics*, the *Eudemian* and the *Nicomachean Ethics* and finally the *Rhetoric* whereas they refer to no other writing of Aristotle's with the exception of the *Topics*, we cannot confidently use this as evidence of an *early* dating (pace Ross, APPA, p. 23) – for it is in the nature of the case that a scientific discussion should import its logical foundations from abroad, whereas a treatise on logic should have small opportunity to refer to the individual sciences. Nevertheless, it is reasonable to suppose that the last years of Aristotle's life were filled by his wide-ranging scientific, historical and literary researches.
3. This use of the expression 'definite' is due to Lorenzen (*Einführung in die operative Logik und Mathematik*, 1955, pp. 5 sq.); it seems to be of importance for philosophy too. Every 'scientific' statement must be 'definite', that is either 'definite qua provable' or 'definite qua refutable'. An interpretation is 'refuted' if we can prove a contradiction between statements in the interpretation itself, or between the interpretation and the text, or between consequences of the interpretation and of the text.

4. (a) ὅταν οὖν ὅροι τρεῖς οὕτως ἔχωσιν πρὸς ἀλλήλους ὥστε τὸν ἔσχατον ἐν ὅλῳ εἶναι τῷ μέσῳ καὶ τὸν μέσον ἐν ὅλῳ τῷ πρώτῳ ἢ εἶναι ἢ μὴ εἶναι, ἀνάγκη τῶν ἄκρων εἶναι συλλογισμὸν τέλειον. (b) καλῶ δὲ μέσον μὲν ὃ καὶ αὐτὸ ἐν ἄλλῳ καὶ ἄλλο ἐν τούτῳ ἐστίν, ὃ καὶ τῇ θέσει γίνεται μέσον. (c) ἄκρα δὲ τὸ αὐτό τε ἐν ἄλλῳ ὂν καὶ ἐν ᾧ ἄλλο ἐστίν (*A* 4, 25b32–37).
5. It is taken in this sense by Kirchmann, *Erläuterungen zu Aristoteles' Organon*, Leipzig, 1877, p. 11 et passim.
6. τὸ δὲ ἐν ὅλῳ εἶναι ἕτερον ἑτέρῳ καὶ τὸ κατὰ παντὸς κατηγορεῖσθαι θατέρου θάτερον ταὐτόν ἐστιν (*APr.* A 1, 24b26–28).
7. ὅρα δὲ πῶς ἡμῖν τὴν ἰδιότητα τοῦ πρώτου σχήματος ἐσήμανεν εἰπὼν "τὸν ἔσχατον ἐν ὅλῳ εἶναι τῷ μέσῳ" ἀντὶ τοῦ τὸν μέσον τοῦ ἐσχάτου παντὸς κατηγορεῖσθαι (*in APr.* 72, 17–20).
8. ταὐτὸν γὰρ τὸ κατὰ παντὸς καὶ τὸ ἐν ὅλῳ μόνῃ τῇ σχέσει διαφέροντα, ὡς εἶναι, ὅταν μὲν ἐκ τοῦ κατὰ παντὸς ποιώμεθα τοὺς συλλογισμούς, μείζονα ὅρον τὸν πρῶτον, ὁμοίως καὶ πρότασιν μείζονα τὴν πρώτην, ὅταν δὲ ἐκ τοῦ ἐν ὅλῳ, τὸν τελευταῖον καὶ τὴν δευτέραν πρότασιν (*ib.* 78, 4–8).
9. δῆλον δὲ καὶ ὅτι ἐν ἅπασι τοῖς σχήμασιν, ὅταν μὴ γίνηται συλλογισμός, κατηγορικῶν μὲν ἢ στερητικῶν ἀμφοτέρων ὄντων τῶν ὅρων οὐδὲν ὅλως γίνεται ἀναγκαῖον, κατηγορικοῦ δὲ καὶ στερητικοῦ, καθόλου ληφθέντος τοῦ στερητικοῦ ἀεὶ γίνεται συλλογισμὸς τοῦ ἐλάττονος ἄκρου πρὸς τὸ μεῖζον.
10. This proposition, which has the form "If not-*A* then *A*" (Not-$p \rightarrow p$), is of course not a contradiction: in propositional logic it is equivalent to *A*, that is here to the proposition "there is a syllogism". However, Aristotle treats a proposition of precisely this form as a contradiction (like "*A and* not-*A*") at *APr. B* 4, 57b12–14. On this cf. Patzig, 'Aristotle and Syllogisms from False Premisses', *Mind* **68** (1959), 186–192 (See Appendix below.).
11. πρὸς μὲν τὴν τοῦ προκειμένου δεῖξίν εἰσιν ἀσυλλόγιστοι, ἄλλο μέντοι τι ἐξ αὐτῶν ἔστι συλλογίσασθαι καὶ δεῖξαι (*in APr.* 109, 10–12).
12. Cf. pp. 44 sq. above.
13. οἷον εἰ τὸ μὲν *A* παντὶ τῷ *B* ἢ τινί, τὸ δὲ *B* μηδενὶ τῷ *Γ*· ἀντιστρεφομένων γὰρ τῶν προτάσεων ἀνάγκη τὸ *Γ* τινὶ τῷ *A* μὴ ὑπάρχειν. ὁμοίως δὲ κἀπὶ τῶν ἑτέρων σχημάτων· ἀεὶ γὰρ γίνεται διὰ τῆς ἀντιστροφῆς συλλογισμός (*A* 7, 29a23–27).
14. The translators generally follow Philoponus: Rolfes translates ἀντιστρεφομένων τῶν προτάσεων "wenn man dann die Sätze umkehrt" and διὰ τῆς ἀντιστροφῆς by "durch Umkehrung" ("immer ergibt sich durch Umkehrung ein Schluss", p. 17); the otherwise excellent translation by A.J. Jenkinson (Oxford, 1928), runs "if the premisses are converted" and "by means of conversion". Łukasiewicz, who translates the whole of *A* 7, 29a19–26, takes διὰ τῆς ἀντιστροφῆς, as I do, to hint at the *proof*, not the *condition*, of the validity of these moods. However, he bestows not one word on the paradox: "whenever a syllogism does not result ... a syllogism always results" (AS, p. 25). Nor does Ross say anything about it in his commentary. The Oxford translation softens the paradox by adding a word which is not in the Greek: "Whenever a *proper* syllogism does not result ...".

   Karl Zell, whom Waitz so sharply criticized ("haud paucis locis veram Aristotelis mentem eum non perspexisse apparet", I, pp. XII–XIII), is to my knowledge the only translator who is right on this point: "so muss notwendig, *da* die Vordersätze sich umkehren *lassen*, *C* einigen *A* nicht zukommen" (*Aristoteles' Werke*, *Organon*, Stuttgart ,1836, p. 157). We shall shortly meet Zell again (below,

n. 20), in connexion with his correct reading of a difficult and generally misunderstood point of Aristotle's doctrine. Zell's translation is not, of course, entirely free from error: but it is at least preferable to Rolfes'.

15. That is, premisses of different *quality*.
16. This section of the *Analytics* has often been wrongly interpreted in the past. Cf. Patzig, *Mind* **68** (1959), 186–192 (See Appendix below.).
17. This argument of Łukasiewicz, which I followed in the first edition of this book, now appears less convincing: for Aristotle may mean that the middle term is that term which the premisses *assert* (rightly or wrongly) to contain one of the outer terms and to be contained in the other. If this is so, then the formulations in *A* 4, 25b32–37 are not incorrect (though they may be misleading) when applied to syllogisms in *Barbara*. It remains true, of course, that they are not valid for the *other* first figure syllogisms.
18. λέγω δὲ μεῖζον μὲν ἄκρον ἐν ᾧ τὸ μέσον ἐστίν, ἔλαττον δὲ τὸ ὑπὸ τὸ μέσον ὄν (*A* 4, 26a21–23).
19. (a) ὅταν δὲ τὸ αὐτὸ τῷ μὲν παντὶ τῷ δὲ μηδενὶ ὑπάρχῃ, ἢ ἑκατέρῳ παντὶ ἢ μηδενί, τὸ μὲν σχῆμα τὸ τοιοῦτον καλῶ δεύτερον, (b) μέσον δὲ ἐν αὐτῷ λέγω τὸ κατηγορούμενον ἀμφοῖν, ἄκρα δὲ καθ' ὧν λέγεται τοῦτο, (c) μεῖζον δὲ ἄκρον τὸ πρὸς τῷ μέσῳ κείμενον, ἔλαττον δὲ τὸ πορρωτέρω τοῦ μέσου. (d) τίθεται δὲ τὸ μέσον ἔξω μὲν τῶν ἄκρων, πρῶτον δὲ τῇ θέσει (*A* 5, 26b34–39).
20. Zell (o.c. p. 128, n. 14) notes on the first half of (c): "This refers to the subsequent lettering of the terms in the order *M*, *N*, *X*". However, Zell falls back on relative extensions to explain (d): "The middle term is "outside": i.e. it is not the case (as in the first figure) that it is contained in the major and contains the minor. It is "the first" since the two other terms are subordinate to it" (p. 141). Kirchmann (o.c. p. 128, n. 5, p. 32) expressly rejects Zell's correct explanation of (c). Cf. further p. 125.
21. And which Aristotle cannot have inserted until after the composition of *A* 5 and *A* 6.
22. (a) ἐὰν δὲ τῷ αὐτῷ τὸ μὲν παντὶ τὸ δὲ μηδενὶ ὑπάρχῃ, ἢ ἄμφω παντὶ ἢ μηδενί, τὸ μὲν σχῆμα τὸ τοιοῦτον καλῶ τρίτον. (b) μέσον δ' ἐν αὐτῷ λέγω καθ' οὗ ἄμφω τὰ κατηγορούμενα, ἄκρα δὲ τὰ κατηγορούμενα, (c) μεῖζον δ' ἄκρον τὸ πορρώτερον τοῦ μέσου, ἔλαττον δὲ τὸ ἐγγύτερον. (d) τίθεται δὲ τὸ μέσον ἔξω μὲν τῶν ἄκρων, ἔσχατον δὲ τῇ θέσει (*A* 6, 28a10–15).
23. (a) ἀλλὰ πρῶτον ληπτέον τὰς δύο προτάσεις, εἶθ' οὕτω διαιρετέον εἰς τοὺς ὅρους, μέσον δὲ θετέον τῶν ὅρων τὸν ἐν ἀμφοτέραις ταῖς προτάσεσι λεγόμενον. (b) ἀνάγκη γὰρ τὸ μέσον ἐν ἀμφοτέραις ὑπάρχειν ἐν ἅπασι τοῖς σχήμασιν. (c) ἐὰν μὲν οὖν κατηγορῇ καὶ κατηγορῆται τὸ μέσον, ἢ αὐτὸ μὲν κατηγορῇ, ἄλλο δὲ ἐκείνου ἀπαρνῆται, τὸ πρῶτον ἔσται σχῆμα. (d) ἐὰν δὲ καὶ κατηγορῇ καὶ ἀπαρνῆται ἀπό τινος, τὸ μέσον· (e) ἐὰν δ' ἄλλα ἐκείνου κατηγορῆται, ἢ τὸ μὲν ἀπαρνῆται τὸ δὲ κατηγορῆται, τὸ ἔσχατον. (f) οὕτω γὰρ εἶχεν ἐν ἑκάστῳ σχήματι τὸ μέσον (*A* 32, 47a36–b6).
24. (g) ὅσα δ' ἐν πλείοσι περαίνεται, τῇ τοῦ μέσου θέσει γνωριοῦμεν τὸ σχῆμα (*A* 32, 47b13–14).
25. It might be supposed that these definitions also cover the syllogisms appended in *A* 7 and *B* 1, which traditional logic places in the fourth figure; this is true for the premisses of *Bamalip* (*BaA* & *CaB*), of *Calemes* (*BaA* & *CeB*) and of *Dimatis* (*BiA* & *CaB*). However, since sentence (c) admits to the first figure only those moods in which the premiss containing the middle term as predicate is *affirmative*, it would not catch *Fesapo* and *Fresison*. It seems likely that when Aristotle came

to write *A* 32 he was only thinking of the four valid moods of *A* 4; I leave open the question whether or not that is evidence that *A* 7 and *B* 1 were written after our passage.

26. εἰ ἀνάγκη μέν τι λαβεῖν πρὸς ἄμφω κοινόν, τοῦτο δ' ἐνδέχεται τριχῶς (ἢ γὰρ τὸ *A* τοῦ *Γ* καὶ τὸ *Γ* τοῦ *B* κατηγορήσαντας, ἢ τὸ *Γ* κατ' ἀμφοῖν, ἢ ἄμφω κατὰ τοῦ *Γ*), ταῦτα δ' ἐστὶ τὰ εἰρημένα σχήματα, φανερὸν ὅτι πάντα συλλογισμὸν ἀνάγκη γίνεσθαι διὰ τούτων τινὸς τῶν σχημάτων (*APr. A* 23, 41a13–18).

27. Perhaps, however, Aristotle wishes to say nothing in this passage as to which of the terms *A* and *B* is to be subject and which predicate of the conclusion of the syllogisms they are used to construct; if that is so, then Aristotle is *right* in saying that the three figures can receive all possible valid syllogisms. But he will then be wrong when (41a16) he identifies *these* figures with those discussed in *A* 4–6: for the latter consistently observe the condition that the outer term first named in the premisses must be predicate of the conclusion.

28. ἐπεὶ δὲ οἱ μὲν καθόλου τῶν συλλογισμῶν εἰσὶν οἱ δὲ κατὰ μέρος, οἱ μὲν καθόλου πάντες αἰεὶ πλείω συλλογίζονται, τῶν δ' ἐν μέρει οἱ μὲν κατηγορικοὶ πλείω, οἱ δ' ἀποφατικοὶ τὸ συμπέρασμα μόνον. αἱ μὲν γὰρ ἄλλαι προτάσεις ἀντιστρέφουσιν, ἡ δὲ στερητικὴ (sc. κατὰ μέρος) οὐκ ἀντιστρέφει. τὸ δὲ συμπέρασμα τὶ κατὰ τινός ἐστιν, ὥσθ' οἱ μὲν ἄλλοι συλλογισμοὶ πλείω συλλογίζονται, οἷον εἰ τὸ *A* δέδεικται παντὶ τῷ *B* ἢ τινί, καὶ τὸ *B* τινὶ τῷ *A* ἀναγκαῖον ὑπάρχειν, καὶ εἰ μηδενὶ τῷ *B* τὸ *A*, οὐδὲ τὸ *B* οὐδενὶ τῷ *A*, τοῦτο δὲ ἕτερον τοῦ ἔμπροσθεν· εἰ δὲ τινὶ μὴ ὑπάρχει, οὐκ ἀνάγκη καὶ τὸ *B* τινὶ τῷ *A* μὴ ὑπάρχειν· ἐνδέχεται γὰρ παντὶ ὑπάρχειν (*APr. B* 1, 53a3–14).

29. According to Apuleius (*Opera*, ed. Thomas, IV, 193, 16–20) the 'subaltern' moods were introduced by Ariston of Alexandria (c. 50 BC), a Peripatetic. Apuleius calls the move "ineptum". Incidentally, Bocheński (FL, p. 161; HFL, p. 140) must be corrected on this point: he has let himself be confused by Apuleius' statement that *three* of Ariston's new moods belong to the *first* figure and *two* to the *second*. He has not taken into account the fact that Apuleius, like Theophrastus, counts the moods of the later fourth figure as moods of the first. In addition to *Barbari* and *Celaront* Ariston constructed *Celantop* (in the *first* figure, according to Apuleius). Therefore Bocheński's 24.273 should read, not "*A* to all *B*; *B* to some *C*; therefore *C* to some *A*", but rather "*A* to no *B*; *B* to all *C*; therefore *C* not to some *A*" – the subaltern form of *Celantes* (cf. Bocheński, FL, p. 116; HFL, p. 101). With this improvement the "riddle of the text is solved" not "only partially", but completely. – Prantl, who here for once follows Apuleius and calls Ariston's move "colossally stupid" (I, p. 557), conjectures the *third* new mood of the first figure to be $AaB \& BaC \rightarrow CiA$, that is the traditional *Bamalip*. But Apuleius himself has already admitted the mood in this, its Theophrastian, form as the second mood of the first figure (cf. I, p. 557 and 366); the moods introduced by Ariston must of course be ones which Apuleius did *not* allow into his system.

30. I suppose here, for the sake of argument only, that such extensional definitions are possible. Cf. pp. 97 sqq.

31. Ross writes: "Thus he (sc. Aristotle) recognizes the validity of all the inferences which later logicians treated as moods of a fourth figure, but he treats them, *more sensibly*, by way of two appendixes to his treatment of the first figure" (APPA, p. 35 – my italics). Aristotle never explicitly states that the five moods in question belong to the first figure; nor, of course, that they belong to an appendix of the first figure. It is not clear from Ross' words whether the "more

sensibly" only reproaches those logicians who introduced a fourth figure, or whether it also applies to those who, like Theophrastus, regarded the moods in question as legitimate members of the first figure (and not as bastard appendages to it). Whereas Überweg (SdL[5], p. 339) thought that Theophrastus was simply working out Aristotle's own suggestions, Prantl (I, p. 367: "a superficial interpretation") and Maier (SdA II, 1, p. 98: "disastrous for the future development") violently attacked the introduction of the five indirect moods into the first figure.

32. Cf. p. 85, n. 17 and p. 127, n. 2.
33. E.g. Zeller, *Die Philosophie der Griechen* II, 2[2], p. 164: "Aristotle does not take explicit note of the fourth case, in which it (sc. the middle term) is subject of the lower and predicate of the higher term; however, we shall blame him the less for this, since such a case cannot in fact occur in a pure and rigorous procedure". Cf. Prantl I, p. 272.
34. Kant, *Die falsche Spitzfindigkeit der vier syllogistischen Figuren*, 1762, § 4: "The sort of inference in this figure is so unnatural ... " etc. Ross, APPA, p. 35: "The fourth figure draws a completely unnatural conclusion where a completely natural conclusion is possible".
35. Prantl (I, p. 367) calls the sixth Theophrastian mood of the first figure (*Celantes* = *Calemes* (IV)) "utterly worthless"; the eight and ninth (*Fapesmo* = *Fesapo* (IV) and *Frisesomorum* = *Fresison* (IV)) "can have ... no importance whatsoever as special moods of argument" (ib.); finally "the whole fourth figure ... must be labelled a vast and pointless game" (I, p. 574).
36. Since the premisses of, e.g. *Bamalip* (*BaA* & *CaB*) are the *converse* of those of *Barbara* (*AaB* & *BaC*), the expressions "first", "last" and "middle term" could be defined by means of a standard formulation in which the variables are connected by relational expressions *converse* to those used by Aristotle (*a*, *e*, *i*, and *o*: "be said of", "belong to" etc.), e.g. "be contained in" or the copula. The standard form of *Bamalip* would then be: "If the *A* is contained in the whole *B*, and the *B* is contained in the whole *C*, then the *C* must be under (ὑπὸ: p. 99) the *A*". It is distressing enough that the conclusion in this formulation must contain the undefined expression "be under", since Aristotle uses "be contained in" only for universal propositions. More important, assuming such a formulation, no distinction could be made between the terms of the first and the fourth figures. Cf. the immediately following text.
37. καλῶ δὲ μέσον μὲν ὃ καὶ αὐτὸ ἐν ἄλλῳ καὶ ἄλλο ἐν τούτῳ ἐστίν ... ἄκρα δὲ τὸ αὐτό τε ἐν ἄλλῳ ὂν καὶ ἐν ᾧ ἄλλο ἐστίν.
38. Cf. also Alexander, in *APr.* 72, 17 sqq.
39. Cf. also Democritus, frag. *B* 26, Diels-Kranz.
40. Thus Alexander says (72, 21–26) that the predicate of a proposition of the form *AaB* is necessarily more extended than its subject, but that this does not hold for propositions of the form *AeB*.
41. Łukasiewicz, AS, p. 32 makes Philoponus the author. The τίνες, however, whom Alexander (75, 11) mentions and criticizes, propounded the same view, and they will have been Alexander's colleagues and contemporaries.
42. B. Einarson, 'On certain mathematical terms in Aristotle's Logic', *AJP* (1936), 33 sqq. and 151 sqq.
43. Iulius Pacius, *In Porphyrii Isagogen et Aristotelis Organum commentarius analyticus*, Genf, 1605, p. 122: "Merito autem has vocat 'figuras': nam per figuras mathematicas declarantur".

REDUCTION AND DEDUCTION

§ 27. Is Aristotle's Syllogistic an Axiomatised Deductive System?

In *A* 4–6 Aristotle discusses fourteen syllogisms belonging to the first, second and third of the traditional figures. The four syllogisms of the first figure he calls perfect, that is self-evident, arguments. The remaining syllogisms are valid, but their validity is not evident. Aristotle therefore shows that they are valid if the first figure syllogisms are valid, that is, true logical theorems. We might describe this procedure in modern terms as a *proof* of the imperfect syllogisms from certain accepted but unproven *axioms* – here the perfect syllogisms. Aristotle was the first to state the properties of such an axiomatic system. He did this in his description of a "demonstrative science" (ἀποδεικτικὴ ἐπιστήμη) in the first chapters of the *Posterior Analytics*: such a system must be founded on premisses which are (a) true, (b) elementary and underived, (c) more clear and more fundamental than the propositions deduced from them, and (d) the grounds of the deduced propositions.[1] Certain axioms must be presupposed, since not everything can be proved and every proof assumes something from which it proceeds.[2] The formal medium of every proof is the syllogism[3], the first figure being particularly scientific.[4]

The thesis that all proofs are syllogistic in form occurs constantly and explicitly in Aristotle: "Proof (ἀπόδειξις) is a certain sort of syllogism" (*APr. A* 4, 25b30); in the sentence "We learn either by induction or by proof" (*APst. A* 18, 81a39–40: εἴπερ μανθάνομεν ἢ ἐπαγωγῇ ἢ ἀποδείξει ...) "syllogism" (συλλογισμός) can be read for "proof" (ἀπόδειξις) – ἅπαντα γὰρ πιστεύομεν ἢ διὰ συλλογισμοῦ ἢ δἰ ἐπαγωγῆς (*APr. B* 23, 68b13–14). Again, "Every proof and every syllogism must come about through one of the three figures" (*APr. A* 23, 41b1 sq.: πᾶσαν ἀπόδειξιν καὶ πάντα συλλογισμὸν ἀνάγκη γίνεσθαι διὰ τριῶν τῶν προειρημένων σχημάτων). In the *Nicomachean Ethics* (*Z* 3, 1139b26–31) Aristotle distinguishes as the two types of διδασκαλία (instruction, teaching) διδασκαλία δι' ἐπαγωγῆς and διδασκαλία συλλογισμῷ.[5] This

division inspired the later dichotomy of sciences into the 'deductive' and the 'inductive'; for our purposes it is important to note that Aristotle defined the deductive sciences as those whose formal medium is the *syllogism.* The disjunction of syllogism and induction is exhaustive: Aristotle concludes, in our passage, that the 'principles', which owe their name to the fact that they cannot be proved syllogistically, must therefore be known by induction. The vision of knowledge as an interplay between sense-perception (or induction) – by which we ascend from the particular to the universal – and the syllogism, which leads us down again from the universal (two reciprocating movements, like exhalation and inhalation) is further developed and justified in the last chapter of the *Posterior Analytics.* It is neither possible nor necessary to give a critical exegesis of this difficult theory here: it should be clear that in Aristotle's view the syllogism is the only medium of proof. His thesis has had a long and influential history: Hegel's apophthegm "Alles Vernünftige ist ein Schluss" (*Logik*, ed. G. Lasson[2], 1934, II, p. 308) has its roots here; J. M. Keynes, who rejects the thesis, cites (*Formal Logic*[4], 1906, p. 387) Whately, Spalding and J.S. Mill as adherents to the view that all correct reasoning can be reduced to syllogistic form.[6]

But how can the syllogisms themselves be proved – quis demonstrabit demonstrationes ipsas? Or, more precisely, how, according to Aristotle, can syllogistic be constructed as an axiomatic system (ἀποδεικτικὴ ἐπιστήμη)? In the case of syllogistic the first figure syllogisms are clearly the propositions *from* which the remaining syllogisms are proved; but according to Aristotle they must also be the propositions *by means of* which they are proved. All the methods used in *A* 4–6 to show the validity of imperfect syllogisms are called by Aristotle, at one time or another, ἀποδείξεις (conversion: *A* 6, 28a28; reductio ad impossibile: *A* 5, 27b3; *A* 6, 28a23; *A* 8, 30a9; ecthesis: *A* 6, 28a23). But an apodeixis is, as we have seen, always a syllogism. The syllogisms should therefore be proved not only *from* but also *by means of* syllogisms.

This is evidently impossible. By means of a syllogism, again according to Aristotle, I can only ever prove the sort of proposition which can appear as the conclusion of a valid syllogism, that is a proposition of the form AxB. A whole syllogism cannot itself be brought into this form: it is an *implication between* such propositions. The proof of a syllogism must plainly be given by means of a proposition of this sort:

> If the propositions *A* and *B* imply a proposition *C*, then, if *D* and *E* imply *A* and *B*, and *C* implies *F*, *D* and *E* imply *F*.

This is obviously not a syllogism. Propositions of this form belong, if they are true, to the so-called *logic of propositions*[7], in which the variables range over *propositions* and not *terms*. We have already met with a formula of this type, on page 111:

> If two propositions imply a third, they also imply any proposition which the third implies.

This is a special case of the preceding law, since in it the relation between *D*, *E* and *A*, *B* is identity (which is stronger than the required implication), while the implication from *C* to *F* still holds.

In *A* 4–6, as we shall see, Aristotle continually uses such theorems of *propositional* logic, and in other passages he explicitly formulates laws containing propositional variables, which he distinguishes expressis verbis from variables which take terms as their values.[8] Thus there is an inconsistency between Aristotle's *doctrine* that every proof must be in syllogistic form and his *practice* of deducing the imperfect from the perfect syllogisms with the help of certain *non-syllogistic* (propositional) laws. However, in his syllogistic theory Aristotle chose an idiom which is peculiarly adapted to keep this difficulty in the background. He generally says, not that the imperfect syllogisms are proved, but that they are "reduced" to perfect syllogisms or that they are "perfected", "brought to perfection", or "completed" by them.[9] In traditional logic these operations are usually collected under the name of reduction. On this Łukasiewicz says: "Aristotle reduces the so-called imperfect syllogisms to the perfect, i.e. to the axioms. Reduction here means proof or deduction of a theorem from the axioms" (AS, p. 74). Aristotle would not agree to this unreservedly; a closer examination of the text shows that he did not mean reduction to be understood as a proof but as a procedure for *transforming* imperfect syllogisms into perfect. We have seen that the perfection of a syllogism depends on certain formal properties of its premisses, properties which only belong to premisses of first figure syllogisms. But a mere proof of a second or third figure syllogism cannot alter its premisses and so do away with its imperfection. If, therefore, Aristotle means by reduction a procedure which turns imperfect syllogisms into perfect

(and evident) ones, it is not legitimate to call reduction a proof. No doubt it makes sense to say that a proof made a proposition evident although it was not so before. But here "evident" is used in that subjective sense which, as we saw (p. 47), Aristotle carefully excludes.

Furthermore, the odd locution "a potential argument" (synonymous with "imperfect argument": *A* 5, 27a2 and *A* 24, 41b33) which, as was shown, properly means "a potentially *perfect* argument" (cf. p. 46), has no clear sense unless we assume that Aristotle intended to state a procedure by which 'actual' syllogisms could be *produced* from these 'potential' ones, i.e. actually evident syllogisms produced from potentially evident ones. And since "evident" as the predicate of a syllogism is defined in such a way that it can only belong to syllogisms of the first figure, Aristotle's 'potential' syllogisms must be those from which first figure syllogisms can be produced – they must be as it were disguised first figure syllogisms.[10] It is easy to 'transform' a second figure into a first figure syllogism; the second premiss need merely be converted. Aristotle always uses this method where it is possible. The text shows that he thought of the matter as the *generation* of a perfect syllogism *from* an imperfect one: *A* 5, 27a12: γεγένηται γὰρ πάλιν τὸ πρῶτον σχῆμα; a36: γίνεται γὰρ συλλογισμὸς διὰ τοῦ πρώτου σχήματος; *A* 6, 28a22: γίνεται γὰρ συλλογισμὸς διὰ τοῦ πρώτου σχήματος – et passim; b34: πάλιν γὰρ ἔσται τὸ πρῶτον σχῆμα τῆς *ΡΣ* προτάσεως ἀντιστραφείσης. Similarly in *A* 7 he maintains that in arguments which are 'perfected' by means of conversion, "the conversion produces the first figure" (29a33–34.: ἡ δὲ ἀντιστροφὴ τὸ πρῶτον ἐποίει σχῆμα), and that through so-called reductio ad impossibile "the first figure comes into being" (29a32).

It is plain that this idiom does not adequately express the thought underlying it. Aristotle may say that an inference of the second figure is 'perfected' by converting one of its premisses and 'reducing' it to a mood of the first figure: nevertheless, it is not the case that one and the same argument was first imperfect and then perfect. Rather, a *new* argument is produced from the imperfect argument, and this new argument is perfect. The arguments of the second and third figures can never, on Aristotle's own principles, remain the same arguments and yet be perfect[11]. For this reason it is misleading to speak of the 'perfecting' of these moods.

The terminology of "reduction", "perfecting" and the "transformation" of "potential" syllogisms into perfect ones is based on a tempting

but misleading model. The validity of an inference cannot depend on the fact that we can generate a perfect syllogism, "produce the first figure", from it. If one syllogism – so the argument runs – can yield another which is obviously valid, then the first argument must be valid too. That is: the raw material of a product can always do whatever the product itself can. Aristotle's idiom, based on the model of production, suppresses the most important factor in the so-called 'reduction' of the imperfect moods to the first figure: the transformation is governed by certain *rules*. We are not 'allowed' to transform an *i*-premiss into its converse *a*-premiss, or simply to convert an *a*-premiss etc. The rules given and invariably used for reduction are such as to guarantee the *logical validity* of the syllogism which is 'transformed' in accordance with them into a mood of the first figure – they are, that is, rules of proof.

The historical fact that Aristotle, understandably overestimating, not the value, but the range of application of his discovery, believed the syllogism to be the only possible form of proof, explains the difficulty he encountered when he tried to prove the syllogisms themselves. His terminology straddles two different ways of evading this difficulty: on the one hand he assumes the first figure syllogisms as undemonstrable and evident axioms and 'reduces' the other moods to them, thus avoiding the word "proof"; on the other, he acknowledges that not all syllogisms can be 'perfected' in this way (notably *Baroco* and *Bocardo*) and that their validity must therefore be *proved*. Aristotle takes great pains to show that these proofs are themselves syllogisms (*A* 7. 29a30–36; cf. *A* 23, 41a21 sqq.). However, for the reasons indicated, that cannot be done: a proof which uses a syllogism is not itself a syllogism, and the conclusion of a syllogism cannot, on Aristotle's definitions, be itself a syllogism but only a proposition in which a subject and a predicate are connected by *a*, *e*, *i* or *o* (or their verbal equivalents).

If these two tendencies of Aristotle's terminology are disregarded (and this may only be done after the reasons behind them have been sufficiently explained) it is certainly possible to regard the different methods of 'reduction' as types of *proof*, and the system as a whole as an axiomatised deductive system (Łukasiewicz, AS, p. 44).[12] The contradictory tendencies of Aristotle's language show that he felt the inconsistency between his theory of proof and his practice in *A* 4–6, but that he could not steel himself to revise the theory. Such a revision might well have led to the

discovery of *propositional* logic as an autonomous and fundamental province of logic alongside the syllogistic logic of predicates. This discovery was in fact made by the Megarians (cf. Bocheński, FL, pp. 133 sqq.), but Aristotle's authority lent such weight to the thesis that every logical deduction must be in syllogistic form that the significance of the Megaro-stoic discoveries was completely misunderstood until the present century.

We may, therefore, without undue violence to Aristotle's thought, construe the different forms of reduction as different *types of proof*. In the following sections the three types (conversion, reductio ad impossibile and ecthesis) will be separately presented. In *A* 4–6 proof by conversion predominates; only in the case of *Baroco* and *Bocardo*, where it is not possible, does reductio ad impossibile occur. Sometimes Aristotle notes that a mood can be proved in more than one way. Here reductio ad impossibile is mentioned as a possible proof in four further cases (*Camestres*, *Darapti*, *Felapton* and *Disamis*). 'Ecthesis' occurs in the assertoric section only as a supplementary method of proof (for *Darapti*, *Datisi* and *Bocardo*, all of the third figure). Since Aristotle later applies a procedure which is the converse of reductio ad impossibile to *all* syllogisms (*APr. B* 2–8), we must suppose that when he wrote *A* 4–6 he was aware that reductio can serve as a proof for all of the second and third figure moods. Thus he rejected the advantage of a *uniform* method for all moods, in favour of proof by conversion which, although not possible in all cases, clearly seemed 'more natural' in his eyes. Hence the frequency of a method of proof is a criterion of the degree of δεικτικὴ δύναμις Aristotle attributed to it; and I shall therefore treat conversion, reductio and ecthesis in that order.

§ 28. Proof by Conversion

If I have proved or assumed it to be evident that a particular syllogism is valid, i.e. that from two premisses *A* and *B* a conclusion *C follows*, then under certain conditions I can prove the validity of other syllogisms too. If a syllogism *X* is valid, then so too is any syllogism *Y*, the premisses of which entail the premisses of *X* and the conclusion of which is entailed by the conclusion of *X*: if we are to prove *Y* from *X*, then the premisses of *Y* must be *at least* as 'strong' as the premisses of *X* and the conclusion of *Y* must be *at most* as 'strong' as the conclusion of *X*. 'Strong' is here

to be defined thus: a proposition p shall be said to be stronger than q if q follows from p and p does not follow from q; p shall be called weaker than q if p follows from q but q does not follow from p; p and q shall be said to be equally strong if p follows from q and q from p. If we neglect the important fact that both the premisses and the conclusion of a syllogism are equally *propositions*, we may illustrate these relationships by comparing the premisses of an argument to a crane and its conclusion to a load which the crane can lift. The premisses of an invalid argument are cranes too weak to raise their loads; premisses from which no conclusion whatever follows are cranes which can lift no load at all – that is to say which are not really cranes (not really premisses) at all. Now if I know that a certain crane can lift a certain load, I also know (a) that this and every other crane of equal strength can lift any lighter load, and (b) that every stronger crane can lift at least the same load. The proofs which Aristotle calls deictic depend on exactly analogous considerations.

Aristotle assumes the perfect first figure syllogisms to be evidently valid. He could now straight away *construct* all syllogisms in which the premisses are 'stronger than' or 'as strong as' the premisses, and the conclusions 'weaker than' or 'as weak as' the conclusions, of the four perfect syllogisms. His actual procedure is somewhat different: he formulates in a row all the sixteen different pairs of premisses in the other two figures which can be constructed by combinations of *a*, *e*, *i*, and *o*, and shows in the case of pairs which yield no conclusion that arguments constructed from these premisses would be invalid. The remaining pairs are either (as in the case of *Cesare*, *A* 5, 27a5–9) immediately transformed into a pair belonging to one of the perfect syllogisms, and then expanded into an argument by the addition of the conclusion of this syllogism; or else they are first fitted out with a conclusion and the validity of the resulting argument is then proved.

Such a procedure plainly requires that it be established what implications hold between propositions of the form AxB and BxA (where x ranges over *a*, *e*, *i*, and *o*). Therefore Aristotle prefaces his systematic discussion in *A* 4–6 by giving rules of conversion to which he later appeals. The rules themselves are discussed in *A* 2: Aristotle first shows that AeB entails BeA (for the proof of this – by ecthesis – cf. § 30) and then proves the further rules $AaB \rightarrow BiA$ and $AiB \rightarrow BiA$ *from* this first rule *by means of* indirect proofs. The proof of $AaB \rightarrow BiA$ runs as follows: "If

the *A* (belongs) to all the *B*, the *B* will belong to some *A*. For if (it belongs) to no (*A*) the *A* will belong to no *B*; but it was assumed that it belongs to all (*B*)" (*A* 2, 25a17–19).[13] Here Aristotle *uses*, but does not explicitly *state*, certain laws of propositional logic. Some parts of the argument must be filled out; in its full form the proof goes: To prove: that *BiA* follows from *AaB*. Proof: suppose that *BiA* does *not* follow from *AaB*; then there must exist at least *one* pair of terms *A* and *B* for which both *AaB* and the negation of *BiA*, that is *BeA*, hold. But *BeA*, as Aristotle has already shown, entails *AeB*. And *AeB* excludes the truth of *AaB*, since *AeB* and *AaB* are contraries, i.e. they cannot both be true together. Therefore: if *BiA* is false, *BeA* is true; but then *AeB* is also true, and hence *AaB* is false. To suppose that the implication $AaB \rightarrow BiA$ does not hold for all terms is therefore to suppose that there are terms *A* and *B* for which a *contradiction* holds. For the assumption that, for certain terms *A* and *B*, *AaB* and *not-BiA*, hence *AaB* and *BeA*, can both be true, is, by virtue of the implications $BeA \rightarrow AeB$ and $AeB \rightarrow \textit{not-}AaB$, equivalent to the contradiction $AaB \,\&\, \textit{not-}AaB$ – and is therefore false. Thus the logical necessity of $AaB \rightarrow BiA$ is demonstrated.

Aristotle uses a theorem of propositional logic that can be formulated thus:

$$[(p \,\&\, {\sim}q) \rightarrow {\sim}p] \rightarrow (p \rightarrow q);$$

in words: "If the conjunction of *p* and the negation of *q* entails the negation of *p*, then *p* entails *q*."

It is readily seen that this theorem is true: We need only note that "$p \rightarrow q$" and "$p \,\&\, {\sim}q$" are each other's *negations*, and that a conjunction is false if one of the conjuncts is false.[14]

The proof of $AiB \rightarrow BiA$ proceeds on similar lines, with the difference that here *AiB* and *AeB* are contradictories, not merely contraries like *AeB* and *AaB*. These rules of conversion are enough to 'reduce' eight of the imperfect syllogisms to perfect ones – to prove the validity of the former from the validity of the latter. (Furthermore, the five moods of the fourth figure can also be derived; but we have already seen that Aristotle pays no attention whatever to the fourth figure in *A* 4–6.)

It will now be a simple matter to understand Aristotle's reductions, despite their concise presentation. Let us begin with the first reduction, that of *Cesare* (II) to *Celarent* (I):

> Let M be said of no N but of all X. Then since the negative proposition converts, the N will (for the future tense cf. p. 18) belong to no M; but it was supposed that M belongs to all X; so that the N (will belong) to no X. For that has already been shown. (*A* 5, 27a5–9).[15]

The first premiss of *Cesare* (II) implies, by the first rule of conversion, the first premiss of *Celarent* (I); the second premisses and the conclusions are identical: *therefore Cesare* is valid. The propositional theorem which this proof is based on is this:

(1) $$[(pq \rightarrow r) \,\&\, (s \rightarrow p)] \rightarrow (sq \rightarrow r)$$

In words: "If the first premiss of a valid syllogism is replaced by a proposition which entails it, then the syllogism remains valid".

We also have:

(2) $$[(pq \rightarrow r) \,\&\, (s \rightarrow q)] \rightarrow (ps \rightarrow r)$$

And the combination of (1) and (2):

(3) $$[(pq \rightarrow r) \,\&\, (s \rightarrow p) \,\&\, (t \rightarrow q)] \rightarrow (st \rightarrow r)$$

Aristotle uses (2) in, for example, the reduction of *Darapti* (III) to *Darii* (I): "If the P and the R belong to all S, then the P will necessarily belong to some R. For, since the affirmative proposition converts, the S will belong to some R; so that, since the P belongs to all S and the S (belongs) to some R, the P necessarily belongs to some R. For a syllogism in the first figure is produced" (*A* 6, 28a18–22).[16] The rule of conversion used here is of course the second, $AaB \rightarrow BiA$.

With the help of (3) and the first and third conversion rules, *Fresison* (IV) can easily be derived from *Ferio*. This was the procedure of later logicians but not of Aristotle; he does not use theorem (3) since by its help only syllogisms of the *fourth* figure can be reduced to those of the first.

The proof for, say, *Camestres* (II) is more difficult: "Again, if the M belongs to all N and to no X, then the X will belong to no N.[17] For if the M belongs to no X, the X (belongs) to no M. But the M belonged to all N. Therefore the X will belong to no N – for the first figure is produced again. But since the negative proposition converts, the N will belong to no X; so that there will be the same syllogism[18]" (*A* 5, 27a9–14).[19] The starting-point here is not the *mood Camestres*, but its premisses alone.

(We have already observed, p. 100, that for many purposes – e.g. definition of the figures, definition of the major and minor terms – Aristotle is interested not in the whole syllogism, but just in the premisses.)

From these premisses, Aristotle asserts, the proposition *XeN* can be inferred. This is proved: the second premiss implies *XeM*; this, together with the first premiss *MaN*, gives the pair *XeM* & *MaN*, from which, by *Celarent* (I), *XeN* follows. In addition to (2) Aristotle uses the law:

$$(pq \rightarrow r) \leftrightarrow (qp \rightarrow r).$$

Except when he wants to give the normal formulation, Aristotle often formulates syllogisms with their premisses transposed: he clearly regards this law to be too obvious to need mentioning. Not until the schoolmen laid down a fixed order for the premisses could the misconception arise that this order had something to do with the validity of the syllogism. Up to now we have proved only the validity of the initial syllogism *MaN* & *MeX*→*XeN*, which is simply *Cesare* (II) with transposed premisses. To reach *Camestres* (II) we must convert the conclusion of *Celarent* by the rule *AeB*→*BeA*. Since the premisses of *Camestres*, as we have shown, imply the premisses of *Celarent*, and the conclusion of *Celarent* implies the conclusion of *Camestres* (*NeX*), *Camestres* is valid. Thus, besides (1) and (2) we use the theorem:

$$(4) \qquad [(p \rightarrow q) \,\&\, (q \rightarrow r) \,\&\, (r \rightarrow s)] \rightarrow (p \rightarrow s)$$

where p stands for the premisses of the imperfect syllogism to be proved; q for the premisses of the perfect syllogism from which it is to be proved; r for the conclusion of the perfect syllogism; and s for the conclusion of the imperfect syllogism.[20]

The proof of *Disamis* (III) proceeds similarly to that of *Camestres* (II), with this important difference: here Aristotle sets out at the start the whole syllogism, premisses *and* conclusion. (The premisses are transposed: cf. p. 60). "If the *R* belongs to all *S* and the *P* to some (*S*), then the *P* must belong to some *R*." (This is not the standard formulation, which would run: "If the *P* belongs to some and the *R* to all *S*, then the *P* must belong to some *R*" cf. p. 103 sq.) Aristotle continues: "For since the affirmative proposition converts, the *S* will belong to some *P*; so that, since the *R* belongs to all *S* and the *S* to some *P*, the *R* will belong to some *P* (by *Darii*); so that the *P* (will belong) to some *R*" (*A* 6, 28b7–11).[21] We can

see that Aristotle's departure from the normal formulation of *Disamis* is deliberate: he transposes the premisses in order to facilitate the transition to *Darii*. The transposition is made tacitly, and rightly so: the mnemonic letter *m*, of traditional logic, which marks transposition of the premisses, does not stand on a level with *p* and *s* (marking conversion of the premisses) insofar as *p* and *s* point to the use of laws of *predicate* logic whereas *m* marks the use of a law of *propositional* logic – our theorem $(pq \to r) \leftrightarrow (qp \to r)$. Aristotle's use of laws of this sort in *A* 4–6 is always tacit, as we have already seen; traditional syllogistic is not consistent here, since it flags the application of *m* but applies other propositional laws tacitly. Aristotle often transposes the premisses of a syllogism. Generally, as here, his text supplies a reason for the transposition; sometimes however it does not (as in the cases of *Felapton* and *Bocardo*, (*A* 6, 28a26–7; b17–19) – cf. Łukasiewicz, AS, pp. 32 sqq.).

As in the case of *Camestres*, the premisses of a perfect syllogism (here *Darii*) are derived from the premisses of the given syllogism (which is at the start only *alleged* to be a syllogism). Thus it is proved that from these premisses the conclusion of *Darii* (*RiP*) can be deduced. So far we have only proved the syllogism $PiS \& RaS \to RiP$. However, since it is established that in syllogisms of the third figure *P* is to signify the major and *R* the minor term, and Aristotle has arbitrarily decided in *A* 4–6 to recognise only those syllogisms in which the major term is predicate of the conclusion, we have not yet reached a well-formed mood of the third figure. Fortunately, however, *RiP* implies *PiR* by the third conversion rule (ὥστε τὸ *Π* τινὶ τῷ *P*: b11), so that we have $PiS \& RaS \to PiR$, a new mood of the third figure – in fact *Disamis*, which was to be proved.

The proofs of the other four syllogisms which Aristotle demonstrates 'directly' by conversion (*Festino* (II), *Felapton* (III), *Datisi* (III), and *Ferison* (III)), offer nothing new. In the following table, I name in each line a perfect first figure syllogism followed by the moods which can be derived from it. The numbers in brackets signify the propositional law on which the proof depends; the letters *a*, *e*, *i*, show which conversion rule is used (*a* stands for $AaB \to BiA$; *e* for $AeB \to BeA$; and *i* for $AiB \to BiA$).

Celarent	: *Cesare* (1) *e*;	*Camestres* (2) *e*;	(4) *e*.
Darii	: *Darapti* (2) *a*;	*Datisi* (2) *i*;	*Disamis* (1) *i*, (4) *i*.
Ferio	: *Festino* (1) *e*;	*Felapton* (2) *a*;	*Ferison* (2) *i*.

It is readily seen that the following syllogisms could also be derived:

from *Celarent*: $BaA \& CeB \rightarrow AeC$ by (4) *e* – this is *Calemes* (IV);
from *Darii*: $BiA \& CaB \rightarrow AiC$ by (4) *i* – *Dimatis* (IV);
from *Ferio*: $BeA \& CaB \rightarrow AoC$ by (1) *e*, (2) *a* – *Fesapo* (IV),
and $BeA \& CiB \rightarrow AoC$ by (1) *e*, (2) *i* – *Fresison* (IV).

It is perhaps surprising that no second or third figure syllogism can be derived from *Barbara*. The reason is of course that no conversion rule has AaB as its consequent: the premisses $AaB \& BaC$ are implied by no formally different pair. However, by rule (4) a new syllogism can be derived from *Barbara* – the syllogism which differs from it only in that its conclusion follows from *Barbara*'s by conversion. This is *Bamalip* (IV), $BaA \& CaB \rightarrow AiC$, which can be deduced by (4) *a* from $CaB \& BaA \rightarrow CaA$ (*Barbara*). We have discussed in § 25 why Aristotle did not deduce or 'reduce' these fourth figure moods in his systematic exposition.

It is plain that the five so-called 'subaltern' moods can also be derived in the same way from the perfect syllogisms, if, besides the conversion rules, the two *rules of subalternation* are admitted ($AaB \rightarrow AiB$ and $AeB \rightarrow AoB$). Aristotle regarded these rules as valid[22], and within his system they are valid, since empty terms (terms which cannot be predicated of any object) are expressly excluded as values of the syllogistic variables (cf. pp. 6 sqq.). However, he does not use these rules to deduce additional syllogisms.[23] If we call the rule $AaB \rightarrow AiB$ *Sa* and the rule $AeB \rightarrow AoB$ *Se*, we can state the possible derivations of the subaltern moods in the following table:

$AaB \& BaC \rightarrow AiC$ (*Barbari*) from *Barbara* (4) *Sa* or *Darii* (2) *Sa*.
$AeB \& BaC \rightarrow AoC$ (*Celaront*) from *Celarent* (4) *Se* or *Ferio* (2) *Se*.
$BeA \& BaC \rightarrow AoC$ (*Cesaro*) from *Celarent* (1) *e*, (3) *Se* or *Ferio* (1) *e*, (2) *Sa*.
$BaA \& BeC \rightarrow AoC$ (*Camestrop*) from *Celarent* (2) *e*, (4) *e*, (4) *Se*.
$BaA \& CeB \rightarrow AoC$ (*Calemop*) from *Celarent* (4) *e*, (4) *Se*.

So far we have discussed the derivations of imperfect syllogisms legitimated by the three rules of *A* 2 (and tacit use of laws of propositional logic) which Aristotle actually undertook; and we have found them all correct. We have seen further that the five moods of the fourth figure

could have been derived by means of the same logical rules; and that a similar 'reduction', or rather deduction, of the so-called 'subaltern' moods is also possible, if we append and employ two rules of subalternation which Aristotle recognised. Thus, while Aristotle derives only eight of the imperfect moods from the four perfect syllogisms, in fact eighteen could be directly deduced by this method. There remain two moods which are recognised by Aristotle but which cannot be derived by any of the procedures we have so far discussed. These are $BaA \& BoC \rightarrow AoC$ (*Baroco* (II)), and $AoB \& CaB \rightarrow AoC$ (*Bocardo* (III)). It is plain that the premisses of these moods cannot imply those of any perfect syllogism: the only first figure premisses yielded by the premisses of *Baroco* are $AiB \& BoC$ (by (1) *a*), and by those of *Bocardo*, $AoB \& BiC$ (by (2) *a*) – and these pairs are premisses of no syllogism, and a fortiori of no perfect syllogism. It is true that, if we appeal to the laws of *obversion* and *contraposition* – which Aristotle never uses and never mentions –, we can win $Ae(not\text{-}B)$, "A belongs to no not-B", from the first premiss of *Baroco* (BaA), and $(not\text{-}B)iC$, "not-B belongs to some C", by obversion from the second (BoC); and that from these propositions AoC can be deduced by *Ferio*. *Bocardo* can be proved by a similar, though more circuitous, route. However, Aristotle ignored negative terms and hence the logical operation of the negation of term-variables. In any case he would not have called this procedure δεικτικῶς ἀνάγειν in view of its obscurity.

It is important to note here that Aristotle himself says that these two moods can also be proved by ecthesis (cf. § 30). Nevertheless he prefers to use a different method of proof, which he clearly regards as the *second best*. This method must now be discussed.[24]

§ 29. Proof by Reductio ad Impossibile

Proof διὰ τοῦ ἀδυνάτου is first mentioned in *A* 5 (27a14–15) as a second way of proving *Camestres*; but Aristotle does not set out the proof here nor give any indication of its character. Contrariwise, when he first actually proves a syllogism by means of reductio ad impossibile, he strangely omits to say that it *is* a proof διὰ τοῦ ἀδυνάτου. The case is that of *Baroco* (II); again Aristotle's remarks are extremely concise:

"If to all the *N* the *M* (belongs), and it does not belong to some *X*, the *N* necessarily does not belong to some *X*. For if it (sc. the *N*) belongs to

all (X) and the M is said of all N, the M must belong to all X. But it was supposed that it (the M) does not belong to some (X)" (A 5, 27a36–b1).[25]

The explanation of this argument must cause particular difficulty to those interpreters who think that Aristotle's methods of 'reduction' are in all cases meant, not to prove that the reduced *syllogism* is *valid*, but rather to show that the *conclusion* of the imperfect syllogism can also be derived by means of a first figure syllogism. Aristotle's proofs of *syllogisms* are then presented as proofs of the truth of the *propositions* which occur as the *conclusions* of these syllogisms. To confound confusion, the proofs are then tacitly treated as *also* demonstrating that the *syllogisms* are *valid*. This is of course quite mistaken: an argument is not shown to be valid by the fact that its conclusion can also be proved in a *different* way. The present case is a perfect counterexample to the false but widespread notion that by proving an *argument* to be invalid we thereby refute the proposition which it was meant to support. It is clear that Maier, for example, makes no distinction between the proof of a syllogism and the proof of its conclusion. Describing the reduction of *Camestres* (II) he writes (SdA II, 1, p. 82): "The proof of the second form of our (second) figure is more trouble-some. First we must convert the negative minor premiss; then, by an argument in the second mood of the first figure, we reach the conclusion 'No N is X'. Only when this too is converted do we have *the proposition which was to be proved* (my italics) 'No X is N'." Maier proceeds in an exactly similar manner when he discusses the proof of *Baroco*, our present concern (o.c., p. 84): "The third mood (*Festino*) can be proved by the conversion of the universal negative major premiss. This is not the case with the fourth mood (*Baroco*) .... Therefore an indirect proof by means of a deductio ad abs. (absurdum) is produced. We suppose that the *proposition to be proved* 'some X is not N is false ...". Compare note 1 on the same page: "The apagogic proof of the fourth form reads thus: Demonstrandum: ἀνάγκη τὸ N τινὶ τῷ Ξ μὴ ὑπάρχειν". This is clearly based on the assumption that the reduction of imperfect syllogisms does not prove the validity of these *syllogisms* once and for all, but gives a procedure for proving either directly or indirectly the conclusion of any imperfect syllogism from a perfect one. It is apparent that Maier's vacillating attitude toward the imperfect syllogisms, which we have already had frequent cause to notice, has its effect here too: he ascribes to them

a certain, but not quite sufficient, logical validity. His suspicion finds its clearest expression in the paragraph summarizing the reduction of the imperfect syllogisms of the second figure: "They (sc. second figure arguments) did in fact stand in need of proof. For they are altogether imperfect: they are not in themselves concludent and their premisses entail nothing as they are originally stated. If they are to acquire *complete syllogistic necessity* (my italics), logical *functions of another sort* must be added to the premisses ..." (SdA II, 1, p. 88). We meet again the notion we have already criticised (cf. p. 94) that a proof can make valid the proposition which it proves; the idea of 'complete' necessity enriches logic by the singular notion of a half, or perhaps three-quarters, necessity. Muddy thought is perfectly matched by muddy expression, when Maier talks of "logical functions of another kind" which must "be added to" the premisses. Überweg too speaks of reductio ad impossibile as though it were meant to demonstrate the conclusion *SoP* of the syllogisms *Baroco* and *Bocardo* (SdL[5], p. 378); and so do Keynes (*Studies and Exercises in Formal Logic*[1], London, 1906, p. 319) and Ross.[26] Łukasiewicz was right to call Maier's interpretation of our passage, which he criticises, the usual explanation.[27] Therefore, before we discuss the passages and our predecessors' interpretations, we must try to find a satisfactory answer to the following question: What is the difference between proving that the conclusion of a syllogism is true, and proving that the syllogism itself is valid?

A proposition q counts as proved if, for example, we have stated a proposition p and a proposition of the form '$p \to q$', which are either axioms of the system within which the proof is being produced or else have already been proved. The conclusion C of a syllogism $A \& B \to C$ can thus be proved if we (a) prove the truth of the proposition $A \& B \to C$ (or, if this is not possible – if the syllogism is invalid – the validity of another syllogism $D \& E \to C$, or of a simple implication $F \to C$), *and* (b) prove the truth of the propositions A and B themselves (or of D and E, or of F). For the syllogism $A \& B \to C$, if it is valid, tells us only that C is true *if* A and B are true; but a proven proposition must be unconditionally true – that is, (in any deductive system) true simply by reason of the axioms of the system. Thus it is not merely a false interpretation, but a logical error, to regard the transition from perfect to imperfect syllogism, which Aristotle calls reduction, as a proof of the *conclusion* of the imperfect syllo-

gism: the conclusion AeC of the syllogism $BeA \& BaC \rightarrow AeC$ (*Cesare*) is *not* proved by the observation that it is *also* the conclusion of the perfect syllogism $AeB \& BaC \rightarrow AeC$ (*Celarent*) – we have still to prove the two propositions AeB and BaC. And conversely, if I can prove the conclusion of a syllogism, that is of no relevance to the validity of the syllogism itself.[28]

By contrast, Aristotle's reduction procedure is perfectly adapted to prove the validity of the 'reduced' syllogism itself. In the place of q in the proof-schema, insert the *whole syllogism* of the form $A \& B \rightarrow C$; in the place of p, the 'perfect' syllogism to which q must be 'reduced'. The perfect syllogisms are evident, and axioms of the system: p thus fulfills the conditions which were set up for the premisses of a proof. We still need the implication $p \rightarrow q$ – and this is proved by the various conversion rules or propositional laws which we make use of.

After these remarks on the fundamental difference between the proof of a conclusion and the proof of a syllogism – a difference ignored in the traditional interpretation of reduction – we return to the text we started from. Let us repeat it:

"If to all the N the M (belongs), and it does not belong to some X, the N necessarily does not belong to some X. For if (the N) belongs to all (X), and the M is said of all N, the M must belong to all X. But it was supposed that (the M) does not belong to some (X)" (A 5, 27a36–b1).[29]

The traditional interpretation goes somehow like this: to be proved: that NoX follows from MaN and MoX. Proof: assume that NoX is false. Then the contradictory of this, NaX, must be true (for *o*- and *a*-propositions about the same terms are contradictory; and of contradictories at least and at most one is true). But if NaX is true, then from the first premiss of our syllogism (MaN) and this proposition we can infer (by *Barbara*) MaX. This in turn is the contradictory of the second premiss of our argument and therefore cannot be true. But since it was validly deduced by *Barbara*, at least one of the premisses which yielded it must be false (for if the conclusion of a valid syllogism is false, at least one of its premisses *must* be false). But since MaN was a premiss of the syllogism we started from, only the assumed proposition NaX can be false. Therefore NoX must be true. Which was to be proved.

The most important objection to this interpretation follows from our previous remarks: it takes Aristotle's argument to be a proof of the *truth*

of the *proposition NoX*, whereas he plainly wants to prove the *validity* of the *syllogism* of which *NoX* is the conclusion. Łukasiewicz (whose criticisms of this interpretation I shall shortly rehearse) does not notice this side of the matter. It is strange too that, although he calls this the usual explanation (AS, p. 54), he does not ask as his enquiry proceeds whether here, as often, it is not possible to give an interpretation which frees Aristotle's text from the errors which cling to the 'usual explanation'. Indeed, on the next page Łukasiewicz refers to what he has just called the usual explanation as "the proof given by Aristotle" (AS, p. 55). He presents his objections to the traditional explanation as objections to Aristotle himself.

What then are the objections Łukasiewicz raises? First and foremost: the second proof fails in the cases in which the premisses of *Baroco* are *false*. But Aristotle himself expressly says that the validity of a syllogism is independent of the truth of its premisses (*APr. B* 2, 53b4–10). The procedure is thus not universally applicable, as the following example (Łukasiewicz') shows:

> If bird belongs to all animals,
> and bird does not belong to some owls,
> then animal does not belong to some owls (AS, p. 55).

This is a syllogism in *Baroco* and must be *true* if *Baroco* is valid. For if a syllogism is valid, whatever values are substituted for its variables the resulting implication must be true. (That is what validity means). But the contradictory of the conclusion, "Animal belongs to all owls", together with the first premiss "Bird belongs to all animals", yields the conclusion "Bird belongs to all owls". This proposition is *true*: reductio ad impossibile is itself impossible.

It may of course be objected to this argument of Łukasiewicz', which Bocheński (FL, p. 89; HFL, p. 77) repeats and subscribes to, that the 'impossible' to which reduction, on the traditional interpretation, leads, is meant to be (as Maier said), not a simple *falsehood*, but a *contradiction* between the second premiss of the original syllogism and the proposition which, as described, is yielded by *Barbara*. In our example too there is just such a contradiction. It could also be pointed out that reductio was clearly in origin an eristic device by means of which an opponent could

be refuted out of his own mouth.[30] Thus one might imagine a man who admits that the propositions "All Christians believe in an after-life" and "Some brave men do not believe in an after-life" are true, but who refuses to recognise that there are any brave men who are not Christians. We could ask such a man if he believed that *Barbara* was a valid mood, and if he agreed, deduce by *Barbara* from "All brave men are Christians" and "All Christians believe in an after-life" the proposition "All brave men believe in an after-life" – which contradicts one of the propositions he *accepts*. Thus we would have proved that the disputed proposition at least follows from propositions held by our opponent. The question whether or not these propositions are *true* remains in the background.

This is precisely the manoeuvre that Aristotle describes in several passages[31] as the "syllogism by way of the impossible" (συλλογισμὸς διὰ τοῦ ἀδυνάτου). This syllogism is one – the most important – of those which he calls "syllogisms from a hypothesis" (συλλογισμοὶ ἐξ ὑποθέσεως). All these arguments begin with an *assumption*: it must be agreed with our partner in the discussion that proposition *A* holds *if B* is proved. The *B* is proved (syllogistically), and thus, according to the agreement, *A* has also been proved. The 'hypothesis' on which the *per impossibile* procedure rests is that *A* be regarded as proved if *Not* (*not-A*) is proved. This is the reason why Aristotle says that in these cases the hypothesis "*need not be expressed since it is so obvious*".[32] Aristotle distinguishes these "syllogisms by way of the impossible" (as a sub-set of hypothetical syllogisms) from "deictic syllogisms" which deduce 'directly' the proposition they are to prove. He is making, that is, a distinction between deictic and per impossibile *syllogisms*. The point to which all this has been leading is this, that a deictic *syllogism* and the deictic *reduction* of such a syllogism are two utterly different things. No more may *syllogisms* per impossibile (also known as apagogic syllogisms) be confused with the apagogic *reduction* of syllogisms. But that is exactly what previous interpreters have done: the traditional interpretation we have sketched treats the reduction of *Baroco* as a *syllogism per impossibile*; in fact it is an *apagogic proof* of a *deictic* syllogism.

Aristotle's text admits a correct interpretation which observes this dissinction. It can indeed be objected that Aristotle never expressly emphatises the distinction; that would have caused him terminological difficulties in view of his unfortunate belief, discussed in § 27, that every proof

must be a syllogism. Nevertheless, he was well aware, as we have seen, that a syllogism and the reduction of a syllogism are fundamentally different. The δεικτικοὶ συλλογισμοί and the συλλογισμοὶ ἐξ ὑποθέσεως and διὰ τοῦ ἀδυνάτου discussed in *APr. A* 23, 29, 44, and *B* 11, are *not* the syllogisms which in *A* 4–6 are reduced 'deictically' or by reductio ad impossibile to perfect syllogisms. For all these syllogisms, including *Baroco* and *Bocardo*, are, according to the characteristics stated in the chapters in question, themselves *deictic*. In *A* 44 Aristotle states emphatically that syllogisms ἐξ ὑποθέσεως and διὰ τοῦ ἀδυνάτου cannot be reduced to the three figures (and hence not to the first figure).[33] Again, he twice says that everything that can be proved by a deictic syllogism can also be proved by a syllogism per impossibile and vice versa.[34] If the *reductions* were counted among the *syllogisms* of which Aristotle is speaking here, this assertion would be simply false, and would contradict his practice of producing a reductio ad impossibile as the *sole* way of proving a syllogism only where a deictic proof by conversion of the premisses is not *possible*. It is no improvement to say, with Ross (APPA, p. 393), that the assertion is only "broadly speaking true". What Aristotle means, he states unequivocally in the text: from the premisses *AeB* and *BiC* I can give a *deictic* proof of *AoC* ("*A* does not belong to some *C*") in *Ferio*; but I can also prove the same proposition (*from the same premisses*) per impossibile, if I conclude from *AaC* (its negation) and *AeB* to *BeC* by *Cesare*, then in view of *BiC* to *not* (*AaC*), and finally, by the 'obvious hypothesis' to *AoC*.

Let us now return for the second time to our text: "If the *M* belongs to all *N* but does not belong to some *X*, then it is *necessary* that the *N* does not belong to some *X*". What is to be proved is clear enough: it is not that the *N* does not belong to some *X* (as in the apagogic *syllogism*) nor yet that the *N* does *necessarily* not belong to some *X* (this would be the conclusion of a modal inference and could never, on Aristotle's view, follow from two assertoric premisses); it is that the *N* necessarily does not belong to some *X if* the premisses are true. That is the claim. Whoever disputes it must maintain that it is possible for the two premisses to be true and the conclusion *NoX* false. *This* is the assertion which Aristotle's argument refutes: Let us suppose that *NoX* is false. Then it can be proved that one of the premisses too *must* be false: *NaX* and *MaN* imply (by *Barbara*) *MaX*. Therefore, since in no case can the two prem-

isses be true and the conclusion false, our opponent's assertion that *NoX* does not follow from the stated premisses is proved false[35] and the validity of *Baroco* established.

I cannot see how this interpretation fails to square with Aristotle's text. No doubt, the traditional interpretation which Łukasiewicz attacks is compatible with the letter of our passage. But our review of the other relevant texts has shown that this reading makes Aristotle not only err, but actually *contradict himself*. In such circumstances (by Überweg's maxim, cf. p. 122) we must always prefer the interpretation which both squares with the text and frees Aristotle from the charge of inconsistency.

What logical laws does Aristotle use in this reduction? Again, as in the case of conversion, he presupposes certain logical principles, some of which belong to predicate and some to propositional logic. He twice uses the fact that *AaB* and *AoB* are contradictories. Here he cannot refer back to introductory remarks in the *Analytics* itself as he did in the case of conversion. He tacitly assumes that the reader knows the relevant passages in *De Interpretatione* (chapters 7 sqq.). Some further notes on what was later called the logical square can be found in book *B* of the *Prior Analytics*: cf. *B* 8, 59b8–11; *B* 11, 61b6, 62a17–19; *B* 15, 63b23–30. However, in our passage all that is supposed is that *AaB* is the *negation* of *AoB* and vice versa. The *propositional* law on which the proof depends is then this: "If from p and the negation of r, *not*-q follows, then r follows from (p and q)". In symbols:

$$(1)\qquad [(p \& \sim r) \rightarrow \sim q] \rightarrow [(p \& q) \rightarrow r]$$

Let the second half of this implication be our syllogism *Baroco*. It may count as proved, according to the principles set out on p. 146, if (a) the antecedent of the implication is proved, and (b) the validity of the implication is established. If we substitute for p, q, and r, *MaN*, *MoX*, and *NoX* in that order, then the antecedent of the implication is *MaN* & *NaX*→*MaX* (by virtue of the equivalences *Not*(*MoX*)=*MaX* and *Not*(*NoX*)=*NaX*; thus the antecedent is a perfect syllogism (*Barbara*), and so evidently true or 'proved' in the stated sense. It is easy to show that the implication itself holds for all values of p, q, and r; we need the 'rule of transposition' $(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$, which Aristotle knows and uses[36], and two further laws of propositional logic. The proof is set out in Whitehead and Russell's *Principia Mathematica* I, p. 118, under 3.37.

That we are not expecting too much of Aristotle in granting him knowledge of this implication, is proved with perfect clarity by the first sentence of his discussion of the so-called conversio syllogismi at *APr. B* 8–10: "If the conclusion is converted (i.e. negated) and one of the premisses is retained, then the other premiss must be abandoned. For if it is true the conclusion too will be true".[37] This formula answers to the propositional law PM 3.37; more precisely, it is a more general form of it, since it also validates:

$$(2) \qquad [(\sim r \& q) \rightarrow \sim p] \rightarrow [(p \& q) \rightarrow r].$$

(This rule is used in the proof of *Bocardo* (III), *A* 6, 28b17–20.) There is admittedly this difference: here Aristotle starts from valid syllogisms and shows that other valid syllogisms can be derived from them by means of 'conversion'. Thus he assumes *Barbara* and derives *Baroco* and *Bocardo* (*B* 8, 59b28–31). The procedure of reduction is the converse of this: *Baroco*, the demonstrandum, is not deduced from *Barbara*; rather *Barbara* is 'developed' from *Baroco* in accordance with a rule which guarantees the validity of all syllogisms from which perfect syllogisms can be gained. The formula behind the 'proofs' in *B* 8–10 should properly be written:

$$(3) \qquad [(p \& q) \rightarrow r] \rightarrow [(p \& \sim r) \rightarrow \sim q].$$

However, (3) can readily be got from (1) by substituting q for $\sim r$ and $\sim q$ for r. (Note also that the equivalence between $(p \& q) \rightarrow r$ and $(p \& \sim r) \rightarrow \sim q$, allows the inversion of the antecedent and the consequent of the implication).

Not merely can *Baroco* and *Bocardo* be derived from *Barbara*: by Aristotle's rule two further syllogisms can be derived from *every* valid syllogism: in *B* 8 Aristotle derives *Festino* (II) and *Disamis* (III) from *Celarent* (I); *Camestres* (II) and *Ferison* (III) from *Darii* (I); and *Cesare* (II) and *Datisi* (III) from *Ferio* (I). Each triad contains one mood from each of the three figures. In *B* 9 Aristotle derives the same triad, this time starting from the moods of the second figure; and in *B* 10 he does it a third time, setting out from the four third figure moods with particular premisses. Only *Darapti* (III) and *Felapton* (II) are wanting: Aristotle was very close to realising that two new moods result from each of them too – the four *subaltern* moods of the first and second figures. From

Darapti ($AaB \& CaB \rightarrow AiC$) we would first get the premisses $AaB \& AeC$, and conclude to the contradictory of the second premiss, CoB. This would be *Cesaro* (II). Secondly, we would get $AeC \& CaB$, with the conclusion AoB; and this would be *Celaront* (I). However, although he did not contest their validity, Aristotle did not recognise these moods in *A* 4–6. For this reason he adds CeB as conclusion to the premisses $AeC \& AaB$, thus constructing *Cesare* (II) which is elsewhere derived from *Datisi* (III). Aristotle can do this only because he has *independent* knowledge that it is possible to deduce CeB from $AeC \& AaB$; the general rule in itself *only* licenses the transition from *Darapti* to *Cesaro*. The same is the case with *Felapton*: according to the rule, $AeB \& CaB \rightarrow AoC$ must first yield $AaC \& AeB \rightarrow CoB$ (*Camestrop* (II)) and then $AaC \& CaB \rightarrow AiB$, which answers to *Barbari* (I). Aristotle passes straight to *Barbara* and *Camestres*: again he is justified by independent knowledge and motivated by the inhospitality of his system toward the subaltern moods. It does not trouble him that *Barbara* has already been derived from *Bocardo* and *Camestres* from *Ferison*, and that *Barbara* and *Camestres* cannot themselves be *transformed back into Felapton*. This may explain why Aristotle does not set out the derivations from *Disamis* and *Bocardo* in *B* 10: in their groups both *Barbara* and *Celarent* reappear in their proper places and with their normal partners from the two other figures: that might have been puzzling. But perhaps that is to give conjecture too much leash.

However, we have certainly established that only prejudice kept Aristotle from a logical discovery which no one, with the exception of Leibniz[38], came so near to until Keynes[39]: the syllogisms can be arranged in sets of three in such a way that the members of each set can all be derived from one another by application of PM, 3.37. Each triad contains one syllogism from each figure; the syllogisms of the later fourth figure can be divided into two further triads. This assumes that the 'subaltern' moods are included. They can be omitted (if the law of subalternation is not admitted), in which case *Felapton* and *Fesapo* (hence *Bamalip*) must also be dropped, since the 'subaltern' moods can be *derived* from them. From every syllogism with n terms, $n-1$ other syllogisms can be derived, since each syllogism has $n-1$ premisses and a new syllogism results if the negation of the conclusion of the initial syllogism is substituted for any one of the premisses. The number of syllogisms in a system must therefore be divisible by n, the number of terms in each syllogism. These are

some elementary facts about *n*-adic syllogistic, which should be obvious without proof. The number of syllogisms in a ternary system must thus always be a multiple of three, and neither Aristotle's 14 syllogisms, nor the traditional 19 form a system. In Aristotle either *Darapti* and *Felapton* are supernumerary, or else they must be accompanied by *Barbari*, *Celaront*, *Cesaro*, and *Camestrop* (bringing the total to 18); traditional logic needs, in addition to these, *Calemop* (IV) to bring the number up to 24, six in each figure. What system is preferred is a matter of taste, provided only that it contains all the syllogisms which can be derived by recognised rules from recognised syllogisms. If, for example, we assume the modern definitions of *a*- and *e*-propositions, which make $a \to i$, $a \to \sim i$, and $e \to o$ *invalid*, then we must admit all the Aristotelian moods except *Felapton* and *Darapti* and add to them *Calemes*, *Dimatis*, and *Fresison* of the fourth figure, making the total 15.

It might be profitable to trace out further the logical structures that are visible here; but we must return to our sheep. It was tempting to observe how Aristotle uses, in *B* 8–10, the law on which the operation he called conversio syllogismi depends. However, the importance of the passage from our point of view is this: it proves that Aristotle actually knows the theorem of propositional logic which he demonstrably used in reduction, and it must convince the reader that 'conversio syllogismi' is simply the *converse* operation to 'reductio syllogismi ad impossibile'. Aristotle admittedly never says this: the connexion was not apparent to him. The text of the *Prior Analytics* as a whole can be compared to an explorer's pioneering reconnaissance of a hitherto untrodden island. This fits well with Aristotle's consciously loose mode of expression, which deliberately leaves room for later precision. Schopenhauer, it is true, castigated any such procedure: anyone following it simply wants "to be able to draw his head from the noose when necessary".[40] This is pretty: but there are cases when boldness destroys what caution might have secured unharmed. Aristotle approached the syllogism from this side and that with the care and dexterity of a funambulist; and this preserved him from a precipitate 'philosophical' appraisal of what he had found. Many neighbouring logical flora escaped his notice, and he missed important items of logical geography: but this is only the reverse side of the preeminence which marks out his text above all later ones. We need only recall Alexander's ingenious but logically grotesque remarks in his commentary on *A* 4, 25b32

(*in APr.* 49, 6–17), where he holds that the first figure is *apodeictic*, the second *dialectic* and the third *sophistic*. (Only the first figure can prove universal affirmatives; the second concludes only to negations and thus belongs to the dialectician whose primary goal is the *contradiction* of his opponent's assertions; the third belongs to the sophists since they love indeterminate propositions – which Aristotle, as is well known, equates with particular propositions – and in fact only particular conclusions occur in the third figure.) Comment is superfluous. Similar examples are ubiquitous, from Alexander to Prantl, whose impatient polemic against all 'trite formalism' which will not harmonize with his 'philosophical', but utterly false, logic, can only make the modern reader smile.

Łukasiewicz is wrong to treat the "usual interpretation" of the passages in *A* 5 and *A* 6 on reductio ad impossibile as if it were Aristotle's own, even if he might justly have blamed Aristotle because his words do not *exclude* such an interpretation. Łukasiewicz expressly recognises the logical validity of the operations which Aristotle calls conversio syllogismi (AS, p. 57). We have ascertained that the reduction of *Baroco* and *Bocardo* exactly corresponds to this procedure. Łukasiewicz is therefore wrong when he says that the "valid proofs" sketched in *B* 8–10 are *replaced* in *A* 5 and 6 by "insufficient demonstrations per impossibile". And hence we need not ask the reason for this replacement. Łukasiewicz thinks the reason was that Aristotle wrongly failed to recognise *arguments ex hypotheseos* as legitimate instruments of proof (AS, p. 57). But in Aristotle's eyes even Łukasiewicz' "insufficient demonstrations per impossibile" must, like all syllogisms per impossibile, have been demonstrations ex hypotheseos. Thus even if Aristotle did in fact feel such an antipathy toward arguments ex hypotheseos, this could not possibly be the reason for the alleged replacement. Aristotle, according to Łukasiewicz (AS, p. 58), "does not understand the nature of hypothetical arguments": Łukasiewicz does not understand what Aristotle says about hypothetical arguments. According to Aristotle one and the same proposition q can be proved both 'simply', that is 'deictically', by proving q itself, and also hypothetically, by proving a proposition p and proceeding by means of an implication $p \rightarrow q$ to q. The implication $p \rightarrow q$ need be neither proved nor evident, in which case q is only hypothetically proved – that is, proved for all who are willing to *grant* the implication. If the implication is not explicitly proved but can be assumed as *evidently valid*,

we have the case presented by reductio ad impossibile. Here the implication which effects the proof has the form (*not-not-q*)→*q*; that *q* follows from *not-not-q*, is an *evident* hypothesis, and its evidence allows us to regard reductio ad impossibile, *hypothetical* though it is, as a *valid* proof of *q*. That is what Aristotle means when he says at *APst. A* 11, 77a22–24, that reductio "assumes the law of excluded middle".[41] Łukasiewicz continues: "The proof of *Baroco* and *Bocardo* by the law of transposition is not reached by an admission or some other hypothesis, but performed by an evident logical law" (AS, p. 58). But Aristotle does not mention an "admission" in connexion with the proofs of *Baroco* and *Bocardo*; and a hypothesis can in his view and in accordance with his principles, sometimes be an *evident* logical law – as it is in reductio ad impossibile.

In sum: Aristotle proves *Baroco* and *Bocardo* by reductio ad impossibile. This procedure was grossly misunderstood by traditional logic, principally because it did not observe that a proof of the *conclusions* of these arguments cannot serve as the required proof of the *syllogisms* themselves. We have shown that reductio ad impossibile and the syllogism per impossibile are two distinct things, and that Aristotle realised this. Reductio ad impossibile is the *converse* operation to that described in *B* 8–10; and it is logically immaculate.

§ 30. Proof by Ecthesis

In comparison with proof by conversion of premisses (or conclusion) and by reductio ad impossibile, the third type of proof mentioned by Aristotle, *ecthesis*, plays only a minor part. Ecthesis is used as a third way of proving *Darapti* (III), and mentioned as a possible way of proving *Datisi* (III) and *Bocardo* (III), where it is a second possibility after reductio and the expressions "ἔκθεσις" and "ἐκτίθεσθαι" do not occur. An ecthetic proof is only set out, again with unwelcome brevity, in the case of *Darapti* (*A* 6, 28a24–26). Outside the systematic chapters *A* 4–6 Aristotle uses ecthesis in his proof of the convertibility of *AeB* at *A* 2, 25a14–17 (cf. p. 138). In his systematic summary in *A* 7, Aristotle only notices proofs by conversion and reductio. Hence it has been concluded (Łukasiewicz, AS, p. 67; Maier, SdA II, 2, p. 147) that he accorded only a restricted value to ecthesis as a method of proof. But the omission may (and does)

stem entirely from the fact that in *A* 7 Aristotle is only interested in the proofs by which imperfect syllogisms are reduced to *perfect* ones. And ecthesis admittedly does not do that. But it cannot be inferred from this that Aristotle "did not recognise it as a rigorous method of proof" (Maier): in his modal logic Aristotle proves two moods by ecthesis *alone*. They are *Baroco* and *Bocardo* with two necessary premisses (*A* 8, 30a 6–16: on this passage cf. below, pp. 165 sqq.). Reductio ad impossibile, customary elsewhere, fails here; the contradictory of the conjectured conclusion of these moods would be the proposition "*A* belongs possibly to all *C*", and this will not combine with either of the initial premisses to yield one of Aristotle's perfect syllogisms nor even one of the syllogisms so far derived.

The details of Aristotle's *modal* logic are not our concern; we shall only draw on it where it seems to help us understand his assertoric logic. In *assertoric* logic at least nothing rests on the rigour of this type of proof: all syllogisms can also be proved in other ways. Here ecthesis is a redundant cook, and unfortunately the broth suffers. Just because ecthesis is dispensable, Aristotle did not spend long in explaining it. His immediate successors, Theophrastus and Eudemus, did not understand it. The most confusing fact of all is that Aristotle uses the same expressions, "ἔκθεσις" and "ἐκτίθεσθαι", to name operations which are obviously quite different. He himself was aware that these operations are different; but he must have thought them in some ways related, or he would not have called them all ἔκθεσις. In the same way Aristotle talks of ἀντιστροφή and ἀντιστρέφειν in at least four different contexts (Ross, APPA, p. 293, distinguishes six different "usages"): (a) the conversion of *propositions* in accordance with the rules stated in § 28, where the 'counterturn' consists in the change of places by subject and predicate. In (b) the so-called conversio syllogismi, the conclusion is 'turned round', that is, (c) it is replaced by its negation (hence Aristotle distinguishes ἐναντίως and ἀντικειμένως ἀντιστρέφειν, *B* 8, 59b9). (d) Two (equivalent) propositions or predicates stand in a relation of ἀντιστροφή if they can change places in an implication without making the implication false.[42] It may seem tempting to take (d) as the basic sense and to derive (a), (b) and (c) from it by *adding further* stipulations. But since Aristotle says nothing about his use of the word "ἀντιστρέφειν" we shall rather suppose that he let himself be guided by its normal meaning, "turn

round", and trusted his readers to draw the information necessary to understand it from the context in which it appeared. (In English too one word can bear the meanings (a) and (c): if I make a statement (say, "All *A* are *B*") and someone says "the reverse is true", he may mean either that in fact all *B* is *A*, or else that the proposition is false and that *AoB* or *AeB* is true.)

What, then, of ecthesis? Can we find here too some connexion between the operations which Aristotle calls by the same name? And does the ordinary Greek use of the word give us any indication *why* he chose to call these operations ἔκθεσις? We must first distinguish the two main meanings which "ἔκθεσις" bears in Aristotle's logic: (1) the proof of a syllogism – the present object of enquiry; (2) the extrication of three terms from the linguistic web of an argument which is to be tested for logical validity or put into normal syllogistic form. Aristotle deals with (2) – in modern terms, the translation of an argument expressed in ordinary language into the symbols of the syllogistic calculus – and in particular with the difficulties and possible sources of error which beset it, in *A* 33–43; and there can be no doubt that he did not confuse this procedure, without which no well-formed syllogism could be constructed, with the proof of a syllogism's *validity*, which he also calls ἔκθεσις. The name "ecthesis" in sense (2) plainly refers to the 'extraction' of the three terms which are to replace the variables from the linguistic trappings of the sentences in which they occur. Sometimes the word is used for the procedure opposite to this, whereby a syllogism is illustrated by having its variables replaced by concrete terms (e.g. *A* 10, 30b31 sqq.; *B* 4, 57a35). The ordinary language sense of "ἔκθεσις" – Herodotus uses it for the exposure of children and Aristotle himself to describe Odysseus' being landed on the shores of Ithaca by the Phaeacians (*Poet.* 24, 1460a36; cf. *Od.* ν 166) – can cover all these cases. It is even closer to the usage we find in Aristotle's *Metaphysics.* There the Platonists' way of setting up the Ideas as things of a particular kind over and above the phenomena is called ἔκθεσις (*Met. A* 9, 992b10; *B* 6, 1003a10; *M* 9, 1086b10). The expression is apposite, given Aristotle's doctrine of the εἶδος as the formal principle immanent in things; the word is at least clearer than "χωρισμός", the expression which is more frequent and has had greater influence on the history of philosophy, but which rather recalls the unbridgeable chasm between the *world* of perception and the *realm* of ideas as a whole; and

it is more exact than "ὑπόστασις" which is not found in Aristotle in this sense and which in any case does not suit the distinction between the Platonic and the Aristotelian theory of ideas. "ἔκθεσις" exactly captures the extrapolation of a thing from its original context, and hence in Aristotle's criticism of Plato the artificial extraction of the εἶδος from its connexion with the individual thing – a connexion which alone makes the thing a thing and the εἶδος the form of a thing.

So far we have found the expression "ἔκθεσις" used in Aristotle's writings in its everyday sense (the landing of Odysseus) and also in three distinct and more technical contexts. "Ecthesis" means: (a) the sometimes difficult operation of extracting from an informal argument the three terms which must stand in place of the variables *A*, *B*, *C* in its formalisation; (b) the eliciting from the set of all terms certain suitable concrete terms which will replace the variables of a syllogism and so make its validity more palpable; and (c) the extrapolation of the Ideas from the individual things of which they are Ideas. It is plain that we are not dealing with special 'senses' of "ἔκθεσις" in each of the cases: the word always bears the same meaning, but takes on a different colouring according to the context in which it appears – just as "daring" has the *same* meaning whether it is applied to a general's strategy, an unusual but striking rhetorical trope, or a far-reaching scientific speculation.

Any interpretation of the method of proof which Aristotle calls ἔκθεσις, must, among other things, be sure to explain why Aristotle calls this method ἔκθεσις.

After these prefatory remarks, let us turn to the texts in which Aristotle describes proof by ecthesis. The most important passage is the proof of *Darapti* (III): "It is also possible to make the proof by reductio ad impossibile and by ecthesis. For if both (sc. *P* and *R*) belong to all *S*, and one of the *S* is taken, say the *N*, then both *P* and *R* will belong to it, so that the *P* will belong to some *R*".[43] It is natural to take Aristotle's proof in this way: if *P* belongs to all *S* and *R* belongs to all *S*, then both also belong to any part extracted from *S*, say *N*. We then have "*P* belongs to all *N*" and "*R* belongs to all *N*", and from these premisses there follows "*P* belongs to some *R*". This, as it stands, would of course be circular: for the mood constructed around *N* is a new syllogism in *Darapti* – and it is *Darapti* that the ecthesis of *N* is meant to prove.

Alexander saw this and therefore rejected the interpretation[44]. The

only way out that he saw was to suppose that we are faced not with a *logical* proof, but with an attempt to convince by appeal to *perception*. N is not meant to be a sub-set of S but an *individual* having the property of S. If we look (in our imaginations?) at N we can see that it also has the properties P and R, and thus that P and R have at least *one* subject in common. This is illustrated by an example: take the syllogism "If animal belongs to all men and rational being belongs to all men, then animal belongs to some rational being": Socrates is a man, the compresence of animality and rationality in him is palpable; thus the syllogism is proved (*in APr.* 100, 1–7). Even if we were willing to regard this as a proof of Alexander's concrete syllogism, it must still be shown that the syllogism yields a true proposition for *any terms whatsoever* – that it is valid. And as Aristotle himself well knew (cf. § 31), concrete examples can refute an alleged syllogism, but they cannot prove it. A proof must therefore stay in the realm of *variables*: and that is enough to rule out Alexander's interpretation. For however much confidence we may have in our perception and imagination, it is simply impossible to think of an individual which belongs to the class S and has the properties P and R unless we replace S, P, and R by concrete terms: and that we are forbidden to do.

The same objection rules out the interpretations of Ross and Maier. Both follow Alexander in supposing that Aristotle must have had some sort of intuitive proof in mind, because the syllogistic proof sketched and rejected by Alexander is obviously circular. "He must, I think, mean to be justifying the conclusion by appealing to something more intuitive than abstract proof – to be calling for an act of imagination in which we conjure up a particular S which is both R and P and can see by imagination rather than by reasoning the possession of the attribute P by one R" (Ross, APPA, p. 32). "Hence we have immediately intuitive evidence that some R is P" (Maier, SdA II, 2, p. 89). Both differ from Alexander in taking the N not as an individual but as a sub-*set* or subordinate *term* of S (Ross, APPA, p. 318; Maier, SdA II, 1, pp. 105; 101, n. 2; 106). But this is no improvement. By no "act of imagination" can we intuit a term which is determined only by the *variables* S, P, and R. Could we "see by imagination" a rod x inches long? At the root of this mistake, in Alexander as in the modern commentators, lies the belief that a proposition which cannot be proved syllogistically cannot be proved *logically* at all,

but must be shown by appeal to perception, by "conjuring up a particular S", and similar desperate expedients. Łukasiewicz (AS, § 19, pp. 59–67) was the first to show that Aristotle, here as ever, argues *logically* and '*abstractly*', using several laws well-known to modern logicians. (As often, he does not expressly formulate them.) Unfortunately, the ecthetic proofs of *Baroco* and *Bocardo* with two necessary premisses, which Aristotle sets out in his modal logic, are relegated to a footnote by Łukasiewicz (AS, p. 59 – they were pointed out to him by Ross) and the passages are not analyzed. In fact these proofs support Łukasiewicz' proposed exegesis far better than the passages he refers to; moreover, they modify his interpretation in some not unimportant points.

All Aristotle's proofs by ecthesis depend on two *logical* laws which serve to show up the characteristics of particular propositions:

(*i*) If A belongs to some B, then there is a term C such that A and B belong to all C.

The term wanted can if necessary be *defined* as the *logical product* of A and B. Let A stand for "Christian" and B for "scientist". The assertion that some scientists are Christian is clearly equivalent to the assertion that there is a class C such that both B and A belong to *all* its elements. The existence of such a class can be known for certain since it can if necessary be *constructed* by forming the logical product of A and B, in our case the class of Christian scientists. In symbols:

$$AiB \rightarrow (\exists C)\,(AaC \,\&\, BaC)$$ [45]

The second law runs parallel:

(*o*) If A does not belong to some B, then there is a term C such that A belongs to no C and B belongs to all C.

In symbols:

$$AoB \rightarrow (\exists C)\,(AeC \,\&\, BaC)$$

If A stands for "commissioned" and B for "officer", then the N.C.O.'s would be the class defined by the term C. Of course other terms could be substituted for C: we have only shown that given any A and B it is always possible to construct a term C – in the first case, $A \& B$, in the second $(not\text{-}A) \& B$.

These equivalences (for the implication signs may plainly be replaced

by equivalence signs) play an important role in modern expositions of syllogistic (e.g. by B. v. Freytag-Löringhoff and Lorenzen); it is interesting to note that Aristotle already used analogous notions. As in our previous cases of non-syllogistic logical rules, Aristotle *uses* these two laws in ecthetic proofs without *introducing* them explicitly. However, it has apparently not been noticed before that in *APr. A* 28, a chapter already referred to (pp. 6–7), there are sentences which come very close to formulating the two logical laws (i) and (o):

"*If we want to prove that one term belongs to some of another, we must consider the terms to all of which each term belongs*: if a term to all of which the one term belongs is identical with a term to all of which the other belongs, then the first term must belong to some of the second."[46]

"*If we want to prove that a term does not belong to some of another term, then we must consider those terms to all of which the first belongs and those to none of which the second belongs.*[47] If these two sets of terms have an element in common, then the first term must not belong to some of the second."[48]

It is clearly stated here that the existence of a middle term C that satisfies the requirements described is a necessary (βλεπτέον: 43b30) and a sufficient (ἀνάγκη: 43b32; 44a1–11) condition for the validity of the propositions AiB or AoB: and this is precisely what is asserted by the equivalences we have just set out. We are evidently dealing with purely logical relations between propositions which contain only variables and logical constants (existential quantifiers, connectives, and the relation symbols a, e, i, and o): a proof based on these laws has nothing to do with Maier's "empiricism" or Ross' "imagination".

We meet the first proof of this sort in chapter A 2 of the *Prior Analytics*. It occurs in the proof of the convertibility of e-propositions, which we have already touched on in our discussion of the rules of conversion (p. 138). We must now attend to its ecthetic part. Aristotle says: "If the A belongs to no B, then the B will belong to no A. For if (the B belongs) to some (A), say C, it will not be true that the A belongs to no B (but this is supposed): for the C is one of the B's" (A 2, 25a15–17)[49]. The proof clearly depends on the supposition that BiA, the negation of the consequent BeA, entails AiB, the negation of the antecedent AeB (cf. pp. 138–139). This part of the proof, the convertibility of BiA to AiB, which has not yet been proved or even assumed, is meant to be clarified by the

reference to C, which is both A and B. Aristotle hardly gives a proof; rather, he suggests how the proof *could* be set up[50] – and the interpretation of his suggestion would be quite impossible did not other passages which we shall shortly cite come to our aid.

I shall now sketch the proof Aristotle suggests for the implication $BiA \rightarrow AiB$. First, we have seen from A 28 that Aristotle knew the following logical law:

$$(i') \qquad BiA \rightarrow (\exists C)\,(BaC \,\&\, AaC).$$

Alexander, in his detailed discussion of this passage (31, 6–34, 22) – one of the finest parts of his commentary – weighed the possibility that Aristotle wanted to conclude from the premisses $AaC \,\&\, BaC$ to AiB by *Darapti*. However, he himself rejected this explanation: it would hardly be commendable if Aristotle here assumed the syllogistic moods which are not to be discussed until later in the treatise[51]; moreover, the standard proof of *Darapti* in A 6 uses the conversion $AaB \rightarrow BiA$ which has not yet been proved. Alexander rightly rebukes Theophrastus (and Eudemus) who objected that Aristotle's proof was *circular*, since he tried to show $AeB \rightarrow BeA$ by means of the implication $AiB \rightarrow BiA$ and then later to prove this by means of the conversion of *e*-propositions: in fact Aristotle is here giving a *different* proof of *i*-conversion from that of the later passage. Alexander, as we have said, thinks that, since the proof is not syllogistical, and therefore not logical, it must be carried out empirically.[52] Since, for the reasons we have stated, we are not satisfied with this expedient, we must show first how the proof can be set up with logical rigour, and then that Aristotle would in fact have set it up in this way. This is not difficult: apart from the equivalence (i'), we only need to appeal to the commutativity of conjunction – $p \,\&\, q$ is equivalent to $q \,\&\, p$ (cf. pp. 59, 141) – and the expanded form of the so-called hypothetical syllogism:

$$[(p \rightarrow q) \,\&\, (q \rightarrow r) \,\&\, (r \rightarrow s)] \rightarrow (p \rightarrow s),$$

which Aristotle uses in the reduction of *Camestres* (cf. p. 141, proposition (4).) We then reach the following proof:

$$\begin{aligned}&([BiA \rightarrow (\exists C)\,(BaC \,\&\, AaC)]\\ &\&\, [(\exists C)\,(BaC \,\&\, AaC) \rightarrow (\exists C)\,(AaC \,\&\, BaC)]\\ &\&\, [(\exists C)\,(AaC \,\&\, BaC) \rightarrow AiB]) \rightarrow (BiA \rightarrow AiB).\end{aligned}$$

Łukasiewicz' proof (AS, pp. 61–62), which takes *nine* steps, uses the same logical laws with the addition of two "rules of existential quantifiers". The proof is thereby rigorously formalised, but for that very reason it departs from what was demonstrably Aristotle's procedure. The passage in *A* 28 which served as our starting point is not adduced by Łukasiewicz. He says that "it is probable that Aristotle intuitively felt the truth of these theses without being able to formulate them explicitly" (AS, p. 61: the 'theses' are our two-sided implication between AiB and $(\exists C)(AaC \& BaC)$); this can now be improved: it is *certain* that Aristotle believed these theses to be true; nor was he unable to formulate them explicitly: he *stated* them, informally but clearly enough.

It is not difficult now to present the ecthetic proof of *Darapti* in a form which both satisfies the requirements of logic and fits Aristotle's intimations. The text reads (I repeat from p. 159): "If P and R belong to all S and one of the S, say N, is taken, then both P and R will belong to it, so that the P will belong to some R" (A 26, 28a24–26). Substituting P, R and N for A, B and C in (i), we get:

$$PiR \leftrightarrow (\exists N)(PaN \& RaN).$$

The premisses of *Darapti* are $PaS \& RaS$. If these premisses imply $(\exists N)(PaN \& RaN)$, then, by (i), they also imply PiR, and *Darapti* is proved. Do we have $(PaS \& RaS) \rightarrow (\exists N)(PaN \& RaN)$? Clearly we do: every subset of S, and hence S itself, can stand for the N whose existence is asserted in the consequent of the implication. The proof of *Darapti* thus has this form:

$$\begin{aligned}&\{[(PaS \& RaS) \rightarrow (\exists N)(PaN \& RaN)]\\ &\& [(\exists N)(PaN \& RaN) \rightarrow PiR]\} \rightarrow [(PaS \& RaS) \rightarrow PiR].\end{aligned}$$

Aristotle uses the equivalence (o) for the ecthetic proof of *Bocardo* (III). Here he is even briefer than in the case of *Darapti*. After the reductio we described in § 29 has been presented, he says: "It is also proved without reductio, if one of the S is taken to which P does not belong".[53] The premisses of *Bocardo* are PoS and RaS. By suitable substitution in (o), $AoB \leftrightarrow (\exists C)(AeC \& BaC)$, we find PoS to imply $(\exists N)(PeN \& SaN)$. The second premiss, RaS, together with SaN gives (by *Barbara*) RaN, and this with PeN gives (by *Felapton*) PoR. This is the outline of the proof

which Łukasiewicz has reconstructed from Aristotle's curt direction "If one of the S is taken to which P does not belong".

In detail, Łukasiewicz assumes: $PoS \leftrightarrow (\exists N)(SaN \& PeN)$, as above; *Barbara* (with transposed premisses) in the form $(SaN \& RaS) \rightarrow RaN$; and *Felapton* (also with transposed premisses) in the form $(RaN \& PeN) \rightarrow PoR$. Applying the so-called synthetic theorem, which Alexander[54] attributes to Aristotle and which he certainly used in his analysis of argument-chains, Łukasiewicz passes from *Barbara* and *Felapton* to the law $(SaN \& RaS \& PeN) \rightarrow PoR$; and thence, by a familiar principle of propositional logic[55], to $(SaN \& PeN) \rightarrow (RaS \rightarrow PoR)$. Since it is legitimate to bind by an existential quantifier any variable which appears free in the antecedent but not in the consequent of an implication, we have:

$$(\exists N)(SaN \& PeN) \rightarrow (RaS \rightarrow PoR).$$

This proposition together with (o) yields, by the hypothetical syllogism, $PoS \rightarrow (RaS \rightarrow PoR)$; and from this, by a further propositional law[56], there results $(PoS \& RaS) \rightarrow PoR$. And thus *Bocardo* is deduced.

It may seem a temerarious or fantastical elucidation to spin such a long proof from such short and Delphic hints; at the very least, is this not to ride roughshod over all philological method? However, it should not be forgotten that we are dealing with a text on *logic*. In many cases there is only *one* interpretation which will satisfy all logical requirements. Such is our passage: granted that we are to prove *Bocardo* on the basis of the equivalence (o), the proof *must* run along the lines we have laid down; and we may trust that Aristotle was thinking in approximately the way Łukasiewicz conjectures – and by "approximately", I mean not that Aristotle had some *similar*, equally exact, proof in mind, but that he had in mind precisely this proof with its logical connexions expressed in that looser manner which was the universal custom among logicians until the middle of the nineteenth century.

The proposed interpretation is confirmed with all the clarity that could be desired by a section from Aristotle's modal logic which Łukasiewicz unfortunately omitted to adduce. The passage is in chapter *A* 8 of the *Prior Analytics*. The context is the proof of *Baroco* and *Bocardo* with necessary premisses and necessary conclusion. Since reductio ad impossibile is not available here for the reasons given on page 157, Aristotle proves both moods by ecthesis. What he says about the two moods with

necessary premisses holds also, of course, for the corresponding non-modal syllogisms.

"In the other cases (of moods with two necessary premisses and necessary conclusion) the necessary conclusion will be proved by conversion in the same way as in the assertoric cases. But in the second figure when the universal premiss is affirmative and the particular negative, the proof (that these moods are valid) will not be the same (as in the assertoric cases); but it is necessary to 'expose' (a part of the subject of each negative premiss) to which (the predicate) in each case does not belong, and to make the conclusion with this term as predicate; for it will be necessary in these cases. But if it is necessary with the 'exposed' term as predicate, (it will) also (be necessary) with some of *that* (term) as predicate (i.e. the term from which the 'exposed' term was 'exposed'): for the 'exposed' term is a part of it. Each of the syllogisms (required for the proof) belongs to the same figure (as the syllogism which is to be proved)."[57]

The most important point for us here is, it will appear, the last sentence. Aristotle's description, as we shall soon see, only fits exactly the proof of *Baroco*; but it is not difficult to make it fit *Bocardo* as well. Let us start with *Baroco*[58]: the premisses of *Baroco* are $BaA \& BoC$. It is asserted that AoC follows from $BaA \& BoC$. To prove this, Aristotle says, we must expose a term from the subject of the *o*-proposition, that is from C, such that C belongs to all of it and B belongs to none of it. This is simple: in virtue of equivalence (o) we have:

$$BoC \rightarrow (\exists D)\,(CaD \& BeD).$$

Aristotle now talks of a *syllogism* (30a10, 13) which we are to construct with D and which gives a conclusion involving the terms D and A. This is clearly only possible if we join the first premiss of *Baroco*, BaA, with BeD to form the pair $BaA \& BeD$ which, by *Camestres* (II), yields AeD. Aristotle says that the requisite syllogism belongs, like *Baroco*, to the second figure (30a13): we cannot doubt that he is thinking *exactly* as we suppose him to be. What holds of *all D*, he continues, must hold at least of *some C*. Hence, supposing the premisses of *Baroco*, AoC holds: and this was to be proved. The last part of the argument too should really be brought into syllogistic form: from $AeD \& CaD$ we *infer*, by *Felapton*, AoC. We see, however, that Aristotle does not present this step in its full syllogistic form – probably because he can hardly presuppose *Felapton*,

a *third* figure mood, in his proof of *Baroco* of the *second* figure.

Let us now compare the proof of *Bocardo* in *A* 8 with that which, following Łukasiewicz, we have elicited from the brief notice in *A* 6. We start with the premisses $AoB \& CaB$. By ecthesis we can replace AoB by $(\exists D)(BaD \& AeD)$. Now, however, Aristotle's description in *A* 8 no longer fits: we cannot infer AoC from AeD as we did in the case of *Baroco*; for D is here a subclass not of C, which is to be the subject of the conclusion, but of the *middle* term of *Bocardo*, B, and what holds of D has no effect on C (although it does of course on B). We must therefore use our own wits in drawing the analogy for our mood *Bocardo* from the proof of *Baroco*. This is no Herculean task: we first join the second premiss, CaB, with the proposition provided by ecthesis, BaD, and conclude, by *Barbara*. to CaD. Then with the other premiss AeD, we construct the premisses of *Felapton*: $AeD \& CaD \rightarrow AoC$.

The last syllogism is clearly the one which Aristotle regards as the '*proof*' of *Bocardo*: it alone belongs to the same figure as its demonstrandum, and Aristotle has expressly demanded this. The reason why he again suppresses the second syllogism (*Barbara*) and replaces it by a tacit transition from CaB to CaD cannot be stated with confidence: perhaps he wanted to use only *one* syllogism in the proof to preserve the parallelism with *Baroco*; perhaps the fateful inclination to treat every proof as a syllogism also played a part; it may well be of relevance that in both cases the relation of a whole to its parts is in question, and that Aristotle regarded it as self-evident that what holds of all of a part must hold of at least some of the whole (in *Baroco*) and that what holds of all the whole must hold of all of every part (in *Bocardo*). At all events, this text could hardly provide stronger confirmation of the substantial correctness of the interpretation we previously proposed. This passage, which Łukasiewicz did not notice, is related to his interpretation of the ecthetic proof of *Bocardo* much as – on happy occasions – a newly found papyrus is to a scholar's conjectural restoration of a lacunose text: it fully confirms the sense of the conjecture and it shows in its divergencies the very traits which have elsewhere been found characteristic of the author.

Finally, we must consider whether our interpretation can explain why Aristotle chose to call this sort of proof ecthesis. I think it can: in the other technical uses of the word terms were 'extracted' from a sentence in ordinary language, or a group of terms was selected from the set of all

possible terms to take the place of syllogistic variables: here a subterm of the given term is put on one side and then operated with. This usage of the word accords with its basic meaning (or, to put it more cautiously, its usual meaning in ordinary language), which we saw Aristotle use in the case of Odysseus' disembarkation, just as well as do the other meanings set out at the beginning of this section.[59]

§ 31. Inconcludent Premiss-pairs

In *A* 4–6 Aristotle not only proves the validity of the fourteen syllogisms of the first, second and third figures which he recognises: he also demonstrates that those pairs of syllogistic propositions which are not premisses of one of these syllogisms cannot occur as premisses of any syllogism whatever. The way he proves this is controversial in a double sense: his interpreters (a) do not agree on what he is trying to do; and (b) entertain logical objections to his method. Before describing Aristotle's proofs of invalidity, we must spend a few words in explaining how he reviews the possible pairs of premisses.

In each figure Aristotle surveys the sixteen pairs of propositions which can be constructed by permutation of the constants *a*, *e*, *i*, and *o*. He always investigates first the pairs consisting of two universal propositions, *aa*, *ea*, *ae*, *ee* (where *ea* stands in the first figure for *AeB & BaC*, in the second for *BeA & BaC* etc.). Then come the pairs constructed from one universal and one particular proposition (*ai*, *ei*, *ao*, *eo*, *oa*, *ia*, *ie*, *oe*); and finally the purely particular pairs (*ii*, *io*, *oi*, *oo*). The order of these three *groups* is constant in all the figures, but the order of the pairs *within* each group is not. In the third group alone do the pairs always occur in the same order, *ii*, *io*, *oi*, *oo*. The changes in order are not due to chance: Aristotle wants to set *at the head of* each group the pairs which are in fact concludent, that is, which are premisses of valid syllogisms.

No previous commentator has asked why Aristotle sets out the 16 pairs of each figure in the particular order which we find in the text. The answer to this question is no doubt of little importance for our understanding of Aristotle's syllogistic. Nevertheless it is not entirely without point: Aristotle never says why he chose the order he did; if it can be shown that in spite of this the order is based on certain *principles*, we shall have further evidence that the text of the *Prior Analytics* is set

against a backcloth of ideas and considerations which pass by the hasty reader unnoticed. If we are punctiliously naive, if, that is, we take nothing at all for granted, then in certain cases we can trace these considerations a good part of the way back to their origins. In the present instance we find that in all probability Aristotle worked on the basis of the following schema:

	a	**b**	**c**	**d**
1	*aa*	*ea*	*ae*	*ee*
2	*ai*	*ei*	*ao*	*eo*
3	*ia*	*oa*	*ie*	*oe*
4	*ii*	*oi*	*io*	*oo*

Row **a** contains the pairs consisting of two affirmative propositions; **b**, those consisting of negative major and an affirmative minor; **c**, an affirmative major and a negative minor; **d**, two negative propositions. Line 1 contains the universal pairs; 2, the pairs in which the minor, and 3, those in which the major, is particular; and 4 those consisting of particular propositions alone.

In the first figure Aristotle begins with the pairs *aa* and *ea* of the first line *because* they are premisses of *valid* moods of the first figure (*Barbara* and *Celarent*). He then shows that nothing can be inferred from *ae* and *ee*. Next, turning to the second group (pairs of different quality), he first stays in the rows which contained the concludent pairs *aa* and *ea* and shows *ai* and *ei* to be premisses of valid moods (*Darii* and *Ferio*); then *ia* and *oa* are eliminated. Aristotle now moves to the right along the same line and proves the inconcludency of *ie* and *oe*; and then of *ao* and *eo*. The pairs of the fourth line are summarily dismissed as useless.

In the second figure the procedure is similar: again, the two pairs *ea* and *ae* in the first line are at the head *because* they are the premisses of the *valid* moods *Cesare* and *Camestres*. Then Aristotle investigates *aa* and *ee*, and shows that in the second figure no conclusion of the required form AxC can be deduced from them. Moving to the second group, again he discusses first the pairs which result from weakening one of the premisses of *Camestres* and *Cesare*: *ei* and *ao* (both are concludent: *Festino* and *Baroco*); *oa* and *ie* follow (both useless as premisses). Next, the remaining pairs of the second line, *eo* and *ai*, and of the third line, *oe* and *ia*, are tested; finally the pairs of the fourth line are summarily shown to be useless.

In the third figure, as in the first, only *aa* and *ea* of line 1 can be used as premisses of valid arguments (*Darapti* and *Felapton*). *Nevertheless* Aristotle does not discuss the pairs of the second group in the same order as he did in the first figure, obvious though this course might seem. It is readily seen why he did not: in row **a** *ai* as well as *aa* and *ia* yields a valid syllogism (*Datisi*); Aristotle therefore at once proceeds to the right along the same line and establishes that *oa* too gives a valid argument. The concludence of *ai* and *ia* suggested that the mood got by interchanging the relational constants in each particular mood was valid: *therefore* Aristotle next investigates *ao*, which, however, cannot serve as a premiss-pair. Thus we meet the only case in *A* 4–6 in which an inconcludent pair is discussed *before* a pair of the same group which does yield a conclusion. This (*ei*) is at once retrieved and is followed by its opposite *ie* (inconcludent). Next to the right in the same line is *oe*, which brings along with it *eo*: both are shown to be useless. Finally the same summary rejection of the pairs of the fourth line.

It is not likely that this transparent pattern stems from pure chance.

The first pair of propositions from which no conclusion of the prescribed form AxC can be inferred – the first pair of *propositions* which is not a pair of *premisses* – faces Aristotle at *A* 4, 26a2–3. It is, as the preceding remarks make clear, the pair *ae* of the first figure, that is $AaB \,\&\, BeC$. How does Aristotle prove that this pair is not a pair of premisses? The text reads:

"(a) If the first follows all the middle, and the middle belongs to none of the last, then there will not be a syllogism of the outer terms[60]; (b) for nothing necessary occurs by their being so; (c) for it is possible that the first belongs to all the last and (also that it belongs) to none (of the last), so that neither the particular nor the universal (judgment) is necessary. (d) But if nothing is necessary because of them, then there will not be a syllogism. (e) Terms for 'belongs to all': animal, man, horse; for 'belongs to none': animal, man, stone."[61]

We have already met proposition (a) in our discussion of Aristotle's definitions of the figures (§ 22, p. 93). Sentence (b), "nothing necessary occurs by their being so", we know (from § 6, p. 20) to be a misleading way of expressing the fact that no conclusion of the required type necessarily *follows*. There we established that 'relative' necessity, which we have here and which answers to a universal quantifier over term variables,

is denied by asserting 'relative' *possibility*. And since the negation of a universal quantifier prefixed to a logical formula consists in the assertion that there are arguments which do *not* satisfy the formula, it is equivalent to an existential quantifier over the same variables followed by the *negation* of the formula. The 'relative' possibility which Aristotle asserts in (c) according to his text is, as it ought to be, the negation of 'relative' necessity; it must therefore, if our interpretation in § 7 is correct, be explicable as a sort of existential quantifier.

After this preface let us turn to the interpretations of the passage offered by our predecessors.

Alexander explains the distinction between pairs which are premisses of valid syllogisms (συζυγίαι συλλογιστικαί) and pairs which cannot serve as premisses (συζυγίαι ἀσυλλόγιστοι) by adducing the conceptual twins, Form and Matter. This does not make things any clearer: valid syllogisms are, according to him, those which "do not alter where they alteration find, and do not infer and prove different things on different occasions but always and for all matter preserve one and the same form in the conclusion" (*in APr.* 52, 19–22). On the other hand, a pair of propositions "which is changed and transformed with its matter, and which on different occasions has different and mutually inconsistent conclusions, is an asyllogistic and unreliable pair of premisses; just as a man who does not stand by his opinions is unreliable" (52, 22–25). In his exegesis of our present passage (55, 10–57, 4), Alexander understands by the 'matter' (ὕλη) the different terms which, according to sentence (e), can be substituted for the variables in the pairs. The premisses in question, he says (55, 22 sqq.), given suitably chosen term-material (that is, given the substitution of suitable terms for the variables in $AaB \& BeC$) in one case *entail* AaC and in another *entail* AeC, propositions which are mutually destructive. Since the conclusion from a genuine pair of premisses must have the *same* form for all concrete terms, our pair of propositions cannot be a pair of premisses.[62]

This explanation of the passage, which Philoponus follows (*in APr.* 74, 30–75, 15), appears at first to be fairly illuminating. It seems to explain the expressions – unusually concise even for Aristotle – "Terms for 'belongs to all': animal, man, horse; for 'belongs to none': animal, man, stone" (25a8–9): the premisses "Animal belongs to all men, man belongs to no horse" can in fact be followed by the 'conclusion' "Animal belongs

to all horses": and similarly "Animal belongs to all men, man belongs to no stone" allow the 'conclusion' "Man belongs to no stone". But it is wrong to say that these propositions can be *deduced* from, or *proved* by, the premisses associated with them, and that the "clearest sign" that the pair is inconcludent is the fact that "the conclusions deduced from them destroy one another": the conclusion simply cannot be 'inferred' from the given propositions – and in any case they do not contradict one another since they are about different terms. To say that the pairs are not "reliable" "because propositions of different forms can be deduced from them" is systematically misleading (cf. p. 188, n. 28): it misrepresents the fact that *no* conclusion *whatever* can be inferred, because the premisses (here *AaB* & *BeC*), given suitable terms, are *consistent with* propositions of the form *AaC* and *AeC*, hence also with *AiC* and *AoC*, and thus with *all* possible forms of propositions. This will shortly be expanded.

In his commentary on the passage Waitz limits himself to the correct observation (at least factually correct: it does not clearly grasp Aristotle's text) that, instead of "there results nothing necessary", Aristotle should have said "there results nothing with necessity" (I, p. 381): "nam non quod sequitur necessarium est, sed necessario consequitur propositionibus concessis". We have seen that Aristotle's terminology is quite consistent: Aristotle recognises 'relatively' and 'absolutely' necessary propositions, and the conclusion of a valid syllogism, since it is 'relatively' necessary, is a *necessary proposition*. Waitz does not explain how Aristotle uses his term triads to refute the concludence of the premisses; however, he finds fault with Aristotle's method because it shows by *example* what can only be proved from the logical relation in which the terms stand to one another: "est enim proprium artis logicae, ut terminorum rationem cognoscat, dum res ignoret." This reproach will meet us often until we reach Łukasiewicz; I shall examine its justification when I discuss Łukasiewicz' comments.

Prantl passes over Aristotle's proofs of inconcludence in silence.

Maier, in his commentary on our passage, repeats both Waitz' reproach and Alexander's error. "The proof (of inconcludence) ... is an empirical one. We meet here for the first time a type of argument which will often recur: terms for 'belong to all' are ...: animal, man, horse; for 'belong to no': animal, man, stone. The following combinations are in view:

All man is animal	All man is animal
No horse is man	No stone is man
All horse is animal	No stone is animal

Thus it is shown by examples that, for the pair in question, both a universal affirmative and a universal negative proposition *can be derived* (my italics) from premisses which, logically speaking, are completely equivalent. And hence it is proved in concreto that a combination of this sort admits no syllogism" (SdA II, 1, pp. 75–76). Maier characterises the procedure thus: "Aristotle's method is empirico-experimental .... He tests and experiments in order to see what in fact *results* (my italics) from the different combinations of premisses" (SdA II, 2, p. 87). The proof is thus "logical empiricism" (SdA II, 1, p. 81) – a phrase implying strange notions both of logic *and* of empiricism.

In his German translation of the *Prior Analytics* Rolfes explains the passage as follows: "All men are animals, no horse is a man, therefore no horse is an animal. All men are animals, no stone is a man, therefore all stones are animals. In both cases a conclusion is inferred from true premisses and in both cases the conclusion is false: it follows that neither a universally affirmative nor a universally negative conclusion is yielded. Similarly no particular conclusion can be inferred ..."[63] I do not understand why Rolfes makes an *e*-proposition from "horse" and "animal", which Aristotle says are terms for 'belong to all', and an *a*-proposition from "stone" and "animal", which are intended to be examples of 'belong to none'. Aristotle never says that in both cases a conclusion is *inferred,* nor does he assert that the conclusion is false. How could a false proposition be inferred from true premisses? Finally, the reader is left in the dark as to the reference of the "similarly" in the last sentence – as indeed he is by the whole note.

Ross' commentary is, as usual, clear and definite. We find a reasonable interpretation which can be squared with the text; then a judgment on the value of the method: "It is noticeable that in this and following chapters, where A. states that a particular combination of premisses yields no conclusion he gives no reason for this, e.g. by pointing out that an undistributed middle or an illicit process is involved; but he often points to an empirical fact which shows that the conclusion follows (a lapsus calami for "no conclusion follows"). E.g. here, instead of giving the

reason why All *B* is *A*, No *C* is *B* yields no conclusion, he simply points to one set of values for *A*, *B*, *C* (animal, horse, man) for which, all *B* being *A* and no *C* being *B*, all *C* is in fact *A*, and to another set of values (animal, man, stone) for which, all *B* being *A* and no *C* being *B*, no *C* is in fact *A*. Since in the one case all *C* is *A*, a negative conclusion cannot be valid; and since in the other case no *C* is *A*, an affirmative conclusion cannot be valid. Therefore there is no valid conclusion (with *C* as subject and *A* as predicate)[64]" (APPA, p. 302). On p. 33, after similar remarks, Ross adds that such a method only shows *that* certain pairs of propositions cannot warrant a conclusion, it does not show *why* they cannot. This is an almost verbal reprise of Überweg's opinion: he thinks the method to be deficient in that "the *ratio cognoscendi* of the invalidity does not correspond with its *ratio essendi*" (SdL[5], p. 348). No fault can be found with Ross' interpretation proper, except that it does not precisely reproduce Aristotle's argument, which the text admittedly offers in very abbreviated form.

In contrast to the interpretations we have so far reviewed, it is noteworthy that Ross does not say, and plainly does not think, that the propositions "Every horse is an animal" and "No stone is an animal" are *inferred* in the examples.[65]

We have surveyed the views of commentators prior to Łukasiewicz whose interpretation I do not want to discuss until the end of this section; we must now try to *understand* Aristotle's proof of the inconcludence of certain pairs of propositions. To this end we must remind ourselves of Aristotle's actual procedure in *A* 4–7 and of the logical problems which must have confronted him. The main question of these chapters is this: how can the multitude of pairs which satisfy the definitions of the syllogistic figures (that is, the pairs of propositions which have just one term in common) be divided into two classes such that the one class contains the premiss pairs of valid syllogisms, and the other all the pairs from which no proposition of the prescribed form connecting the two outer terms can follow? Aristotle clearly obtains the initial class by permuting the 16 possible combinations of propositions from *aa* to *oo* in each figure, as we described at the beginning of this section. He only recognises three figures, therefore he only has 3×16 or 48 such pairs, whereas in fact there are 64. For the sake of brevity, I shall call each of the 48 combinations which Aristotle constructs and investigates a *pair*; a pair which can serve as the

premisses of a syllogism which both is valid and is recognised by Aristotle as valid I call for short a *premiss-pair*; finally, a pair which is not a premiss-pair is to be called an *inconcludent pair*. These three groups answer to those which Alexander calls συζυγία (team of propositions), συζυγία συλλογιστική and συζυγία ἀσυλλόγιστος (52, 16 sqq.). Aristotle passes from figure to figure, reviewing in turn all the possible pairs in the three figures he recognised. He shows that a pair is a premiss-pair, or belongs to the class of premiss-pairs, either (in the first figure) by pointing out that it *evidently* yields a conclusion of the prescribed form, AxC, or else (in the second and third figures) by giving a *proof* which reduces it to a premiss-pair of an evident, or perfect, argument. Often he first adds a conclusion to the pair and then ensures the correctness of the adjunction by logical proof. The preceding sections have dealt with this in detail.

Aristotle could now simply define the class of inconcludent pairs as those 34 pairs for which such a proof has *not* been given. This would mean to assume that where Aristotle has not given a proof of membership of the class of premiss-pairs, no such proof *can* be given; this assumption Aristotle could not and would not make. Therefore, he proves not only membership of the class of premiss-pairs, but also membership of the class of inconcludent pairs.

How can such a proof be given? It is clear that the class of premiss-pairs and the class of inconcludent pairs together exhaust the set of pairs. If p is a pair and p is not a premiss-pair, then p must be an inconcludent pair. This suggests that we define the class of inconcludent pairs by the *negation* of the property which serves to determine the class of premiss-pairs. What is this property? Plainly, that from them at least *one* conclusion of the form AaC, AeC, AiC, or AoC *follows*; that is, that from the pair either for all terms AaC follows, or for all terms AeC follows, etc. We must now express this more precisely:

Let p be one of the 48 Aristotelian pairs, $AaB \& BaC \ldots AoB \& CoB$; let P be the class of premiss-pairs; and I the class of inconcludent pairs.

What then are the necessary and sufficient conditions for p's being a P (a particular pair's being a premiss pair)? Obviously, they are these:

$$p \varepsilon P^{66} \leftrightarrow [(A, B, C)\,(p \rightarrow AaC) \text{ or } (A, B, C)\,(p \rightarrow AeC) \text{ or } (A, B, C)\,(p \rightarrow AiC) \text{ or } (A, B, C)\,(p \rightarrow AoC)].$$

To give the necessary and sufficient conditions for a p's being an I (a

pair's belonging to the class of inconcludent pairs) we must construct the *negation* of this expression. Our formula is an alternation of propositions all of the variables of which are bound by universal quantifiers (for variables other than A, B, and C cannot occur in p if p is a pair in the sense defined). The negation of a proposition of the form "p or q or ... or r", is, according to one of de Morgan's laws[67], "*not-p* and *not-q* and ... and *not-r*". Thus in place of "or" in our formula we must read "and", and in place of $(A, B, C)\,(p \rightarrow AaB)$ etc. we must read their negations. On the principle that one counterexample is enough to refute a universal proposition, the negation of $(A, B, C)\,(p \rightarrow AaC)$ is the assertion that there are terms A, B, C for which $p \rightarrow AaC$ does not hold. We have already seen that the negation of "$p \rightarrow q$" is "p and *not-q*" (PM, 4.61). The negation of $(A, B, C)\,(p \rightarrow AaC)$ is therefore $(\exists A, B, C)\,(p \,\&\, AoC)$ (since AoC is the *negation* of AaC). Thus the negation of the *whole* expression which we constructed as equivalent to $p \varepsilon P$ has the form:

$$[(\exists A, B, C)\,(p \,\&\, AoC) \text{ and } (\exists A, B, C)\,(p \,\&\, AiC) \text{ and } (\exists A, B, C)\,(p \,\&\, AeC) \text{ and } (\exists A, B, C)\,(p \,\&\, AaC)];$$

and this is equivalent to saying that p belongs to the class I, or that p is an inconcludent pair, or that $p \varepsilon I$.

This assertion is clearly *proved* if (1) I can give a term triad which both (a) makes p true and (b) makes a proposition of the form AaC true; and (2) I can give a *different* triad which fulfills (a) as above and which also (b′) makes a proposition of the form AeC true. For since AoC follows from AeC, and AiC, within Aristotle's system, follows from AaC, these two triads will *also* be evidence for the propositions $(\exists A, B, C)$ $(p \,\&\, AoC)$ and $(\exists A, B, C)\,(p \,\&\, AiC)$, which are the remaining requirements for the refutation of $p \varepsilon P$ and hence for the proof of $p \varepsilon I$. Quite evidently this is exactly what Aristotle is saying in our passage. He is enquiring whether $AaB \,\&\, BeC$ is a premiss-pair or an inconcludent pair. To *justify* his assertion that it is *not* a premiss-pair he asserts that from the pair neither an *a*- nor an *e*- nor an *i*- nor an *o*-proposition follows; that therefore the conditions for $p \varepsilon P$ are not satisfied. And to *prove* this assertion he gives two triads[68] such that their terms, substituted for A, B, C, (a) in both cases make AaB and BeC true, and (b) in the first case satisfy AaC and (b′) in the second case satisfy AeC. Thus the *existence* of triads which satisfy the conditions for $(AaB \,\&\, BeC)\,\varepsilon I$ is demonstrated.

We can readily see from this discussion why Alexander (*in APr.* 89, 31–90, 27) was right to reject Herminus' assertion that to prove $p\varepsilon I$ it is enough to find two triads such that one satisfies $p\,\&\,AaC$ and the other $p\,\&\,AoC$. For in this case we have only refuted the assertion that p entails a conclusion of the form AeC or AoC or AaC. AiC might still follow. It is possible to find triads which satisfy the premisses of *Darii* (I) and the terms A and C of which can be joined to make true propositions of the form AaC or AoC.[69] In fact it is the case that all terms which satisfy *Darii* always satisfy either AaC or AoC as well. Only AeC is excluded: there is no triad which satisfies *Darii*'s premisses and also AeC. And that is why the inference to the negation of AeC, AiC, is *always* valid.

We have now given sufficient attention to Aristotle's usual method of proving that a pair is inconcludent. However, he sometimes uses a different method, which has a certain resemblance to his reduction of *valid* moods to moods already proved. In these cases he does not state term triads, but says that the inconcludence of one pair has already been proved and that if this pair is inconcludent, the pair under consideration cannot be a premiss-pair. This method only appears where Aristotle has certain *difficulties* with his usual procedure.

These difficulties stem from the fact that a true proposition of the form AiC or AoC neither entails nor excludes the truth of AaC or AeC. Aristotle later calls this feature of these premisses "the indefinite" and he uses it in his proof of their invalidity. Because of 'the indefinite' it is sometimes not possible to find two term triads of the required sort: we cannot, for example, find triads which satisfy *ao* in the first figure without also satisfying *ae*. This example is in fact the first such case which Aristotle discusses (*A* 4, 26a39–b10). The difficulty at first misleads him into making a *mistake*; later he recognises that 'the indefinite' itself can serve as *part of the proof*. He says:

"When (in the first figure) the major premiss is universally affirmative or negative and the minor is particularly negative (i.e. he is dealing with *ao* and *eo* in the first figure), there will not be a syllogism; for example, if the A belongs to all B and the B does not belong to some C ...: for the first may both follow all and follow none of the term (C) to some of which the middle does not belong.[70] Let us suppose the terms: animal, man, white. Then let us take some of the white (things) of which man is not said, say, swan and snow. Animal is said of all the one (swan) and of

none of the other (snow), so that there will not be a syllogism."[71]

Aristotle begins here in the usual manner: he asserts that there are triads such that in the one case *AaB*, *BoC* and *AaC* are true, and in the other (for different *A*, *B*, *C*) *AaB*, *BoC* and *AeC* are true. Terms for the first case are readily found: animal, man, and mammal would satisfy the conditions. For in fact animal belongs to all men, man does not belong to some mammals and animal belongs to all mammals. But it is *impossible* to find terms for *AeC* that also satisfy *AaB* and *BoC*, if *BoC* is required to mean "*B* does not belong to some *C*, but it *does* belong to some *other C*". For *AaB* and *BiC* together *entail AiC* (by *Darii*), and *AeC* cannot therefore hold.

We can remove this difficulty by pointing out, as Aristotle himself later admitted, that *BoC* does not *have* to mean that *BiC* holds too; rather, if *BeC* is true then so is *BoC*. Here, however, Aristotle does not follow this path. He plainly wanted to make his proof immediately convincing by granting to the propositions he uses only the meaning which they bear in *ordinary language*. In ordinary language of course a proposition of the form "*A* does not belong to some *B*" almost always means that some *B*'s are *A*. (This, however, is superficial: even in ordinary language *AoB* cannot *mean* that *AiB* holds too. But *AoB* is generally only *used* in situations in which *AiB* holds, and ordinary language has a tendency to count the situation in which a proposition is normally *used* as a part of its *meaning*.) Aristotle therefore tries to reach *AaC* and *AeC* by some *other* route. He offers the triad animal, man, white. This triad obviously satisfies *AaB* & *BoC*; it also satisfies *AiC* and *AoC*. We have thus far shown that from *AaB* & *BoC* no proposition of the form *AeC* or *AaC* can follow. To exclude the remaining conclusions, *AiC* and *AoC*, we must find triads which satisfy both the pair and also *AeC* or *AaC*. Aristotle wrongly thinks that this demand is fulfilled by the triads animal, man, swan and animal, man, snow *without* depriving the premiss *BoC* of its meaning *BoC* & *BiC*, and that our difficulty is eliminated. The logical error is this: Aristotle thinks that we can substitute "swan" and "snow" for "white" in the second premiss of the initial pair, "Man does not belong to some white things", without changing the quantity of the proposition[72]; for in fact swans and snow are subsets of white things and therefore 'some white things'. "Some white things", however, is not the *same* term as "white"; and the second proposition after substitution would have

the form "Man belongs to *none* of some white things". "Swan" and "snow" are thus not substituted for C in the initial example but for "some C": the proposition, which before had the form BoC, must now have the form Be (*some*-C). The difficulty is not, and cannot be, avoided by this manoeuvre.

Maier interprets the text in this way, but he does not raise any objections to the correctness of the procedure (SdA II, 2, p. 87). Ross offers a different explanation: "The fact that, all men being animals, and some white things not being men, some white things are animals and some are not, shows that premisses of the form AO do not warrant a universal conclusion; but it does not show that a particular conclusion cannot be drawn. Therefore here A. falls back on a new type of proof. Within the class of white things that are not men we can find a part A, e.g. swans, none of whose members are (and *a fortiori* some of whose members are not) men, and all are animals; and another part none of whose members are (and therefore *a fortiori* some of whose members are not) men, and *none* are animals. If the original premisses (All men are animals, Some white things are not men) warranted the conclusion Some white things are not animals, then equally All men are animals, Some swans are not men would warrant the conclusion Some swans are not animals; but all are. And if the original premisses warranted the conclusion Some white things *are* animals, then equally All men are animals, Some snow is not a man, would warrant the conclusion Some snow is an animal; but no snow is. Therefore the original premisses prove nothing." (APPA, p. 304).

This is a good argument: but it does not reproduce Aristotle's train of thought. It would be impossible to see why (apart from carelessness) Aristotle should begin with a triad (animal, white, man) which can yield only *particular* propositions of the form AxC and should then turn to a subclass of one of the terms, C, of this triad and construct with this the required conclusions AaC and AeC. For he could have introduced the triads animal, man, swan and animal, man, snow *straight away* as the two triads which can prove the inconcludence of $AaB \& BoC$. The salient point is of course this: in both these cases Aristotle gets true propositions of the form BeC and not the desired BoC, and his aim is precisely to *avoid* the argument a fortiori which Ross uses twice. This is why he begins, not with these two triads, but with the initial triads animal, man, white for which BiC as well as BoC is true, and tries to retain this property by incor-

rectly substituting "swan" and "snow" for "white" in the second premiss.

However, Aristotle goes on to give a second, and this time correct, proof that *ao* in the first figure belongs to the class of inconcludent pairs:

"Again, since '*B* does not belong to some *C*' is indefinite, and it is true both if (*B*) belongs to *no* (*C*) and if it does *not* belong to *all*, and if we take terms such that it belongs to none then there is no syllogism (for this has already been shown): then it is evident that there will be no syllogism if the terms are related in this way; for (otherwise) there would have been one in the other case too."[73]

Here Aristotle formulates and uses the following rule of propositional logic:

If p and q do not imply r and q implies s, then p and s do not imply r.[74]

The first member of the pairs *AaB* & *BeC* and *AaB* & *BoC* are identical, and *BeC* entails *BoC*. *AaB* & *BeC* is thus, in the sense we defined, the 'stronger' pair. Aristotle reasons by reflecting that nothing can follow from weaker premisses which does not follow from stronger; or, to hold by our simile (p. 138), a weaker crane cannot possibly lift a load which a stronger one cannot move. This principle can of course always be applied where it is possible to derive one pair by conversion or subalternation from another which has already been proved inconcludent. If I have proved a pair inconcludent, then I have also proved all other pairs inconcludent which I can logically *derive* from the first pair.

Aristotle did not work this principle up into a system, as he did the reduction of the valid moods or premiss-pairs to the moods or premiss-pairs of the first figure. The reason for this was plainly that, although he recognised certain moods as 'evidently valid' and hence certain pairs as evident premiss-pairs, he would not, and according to his principles could not, mark out any of the inconcludent pairs as 'evidently' inconcludent. Therefore he stood by what he plainly thought was the most *transparent* method of proof, reference to term triads satisfying the stated conditions, and never used the method "be means of the indefinite" (ἐκ τοῦ ἀδιορίστου) which we have just discussed, unless – as in the cases of *ao* and *eo* in the first, *ai* and *eo* in the second, and *ao* and *eo* in the third figure – he was forced to it by the fact that no terms could be found for one of the two triads which did not *also* satisfy the *universal* form of the second premiss.

The texts of all these passages make it quite clear that it was just this difficulty which induced Aristotle to fall back on the law we have quoted. It is enough if we quote the proof of the inconcludence of *eo* in the second figure:

"Let both premisses be negative, and let the major be universal; i.e. let the M belong to no N and not belong to some X. It is possible that the N belongs to all and to no X. Terms for 'belong to none': black, snow, animal." (In fact black belongs to no snow and does not belong to some animals, and snow belongs to no animals. In the second figure Aristotle always names the middle term *first*.) "Terms for 'belong to all' cannot be found, *if the M belongs to some X and does not* (*belong*) *to some* (*other X*). For if the N (belongs) to all X and the M (belongs) to no N, the M will belong to no X;" (this could be proved by *Celarent* with transposed premisses) "but it was supposed that (the M) belongs to some (X). *Therefore it is not possible to find terms in this way, but it must be proved by means of the indefinite.*"[75]

Łukasiewicz, in his description of the proofs of inconcludence under the title "The rejected forms" (AS, pp. 67–72), deals with both Aristotle's methods of proof. His exposition diverges from Aristotle's text in that according to him Aristotle wants to prove, not the *inconcludence* of certain *pairs*, but the *invalidity* of certain *syllogisms*. The form of Aristotle's argument is, as we have seen, this: from $AaB \& BeC$ nothing follows, that is neither AaC or AeC nor AiC nor AoC follows; for it is possible to find two term triads which satisfy $AaB \& BeC \& AeC$ and $AaB \& BeC \& AaC$. This is quite different from arguing that the *inferences* $AaB \& BeC \rightarrow AaC$, $AaB \& BeC \rightarrow AeC$, $AaB \& BeC \rightarrow AiC$, $AaB \& BeC \rightarrow AoC$ are all *invalid*.

Łukasiewicz puts great value on the *second* of Aristotle's procedures for proving inconcludence. The propositional law which lies behind it he calls a "rule of rejection"; it is analogous in some respects to a "rule of assertion". The *rule of rejection* runs: "If the implication "If α, then β" is asserted but its consequent β is rejected, then its antecedent α must be rejected too" (AS, p. 71). The corresponding *rule of assertion* would read: "If the implication "If α, then β" is asserted and its antecedent α is asserted, then its consequent β must also be asserted" (cf. AS, pp. 81, 88). In our case the rule of rejection would have to be formulated like this: "If syllogism β *were* valid if syllogism α is, and syllogism β is re-

jected, then syllogism α must be rejected too." To get back to Aristotle, we would have to change the formulation yet again; it would have to go: "If it is demonstrable that if a pair p is a premiss pair then p' is also a premiss pair, and it has been shown that p' is an inconcludent pair, then it has been shown that p is an inconcludent pair." This rule can easily be derived from the propositional theorem:

$$\{[(\text{Not: }(p\,\&\,q\rightarrow r)]\,\&\,(p\rightarrow s)\,\&\,(q\rightarrow t)\}\rightarrow[\text{Not: }(s\,\&\,t\rightarrow r)].$$

We have seen, however, that Aristotle prefers to prove inconcludence *immediately* by stating suitable term triads, and that he only uses the method of reduction when this procedure gives rise to the *difficulties* we have described.

Łukasiewicz recognised the method of proof by stating term triads to be "correct", but he adds that "it introduces terms and propositions into logic not germane to it. "Man" and "animal" are not logical terms, and the proposition "All men are animals" is not a logical thesis" (AS, p. 72). Instead of this Łukasiewicz proposes to prove the invalidity of syllogisms in precisely the way Aristotle sets about proving their validity. Aristotle assumed two (or four) moods as axioms and *deduced* the remaining valid moods from them. Here, therefore, we can simply assume as axiomatic the invalidity of certain moods (better: the inconcludence of certain pairs) and then *prove* by means of our logical laws the invalidity of the remaining invalid syllogisms (the inconcludence of the remaining inconcludent pairs). Łukasiewicz has shown that only *two* syllogisms or pairs need be rejected axiomatically ($BaC\,\&\,BaA\rightarrow AiC$ and $BeA\,\&\,BeC\rightarrow AiC$); all the other invalid arguments (inconcludent pairs) can then be *proved* invalid (inconcludent). However, convincing though this sounds at first, it blurs an important distinction – or at least a distinction which would have precluded Aristotle from admitting any such assimilation of his method of rejection to his proof by reduction. In Aristotle's view, we can assume 'axiomatically' that *Barbara* and *Celarent* are valid *only* because they possess certain well-defined properties which make them perfect or *evident*. But what definite sense can be given to the assertion that the two 'syllogisms' of the second figure are *evidently* invalid? To say that these 'syllogisms' are invalid (or that these pairs are inconcludent) is, we have seen, *equivalent* to saying that there are *term triads* which have the

properties requisite for rejection. And this assertion of existence can only be evident if, as it were, I actually have my hands on such triads; the evidence of the axioms could therefore only be assured by using the very procedure which Łukasiewicz introduced the axioms to avoid.

We might still object, as Ross and Überweg do, that Aristotle's argument only gives the 'that' and not the 'why' of invalidity. Ross would prefer certain formal laws from which the invalidity of invalid syllogisms or the inconcludence of inconcludent pairs could be read off; Überweg holds the "intuitive conviction produced by comparing circles" to be more probative. However, the fundamental reason why, say, *BaA* & *BaC* is inconcludent lies in the fact that for any terms *A* and *C* there is *always* a term *B* which will make *BaA* and *BaC* true. If necessary *B* can be defined as the *logical sum* of *A* and *C*; *B* would then determine the class of all individuals which are either *A* or *C*.

Similarly for any terms *A* and *C* there is always a term which satisfies *BeA* & *BeC*. This could be *constructed*, again, by forming the logical product of "*not-A*" and "*not-C*", that is the term "(*not-A*) and (*not-C*)". These logical considerations may have made Aristotle confident that for each inconcludent pair he could *find* the triad necessary to prove its inconcludence. Rather than construct a middle term he prefers to introduce a term which is named by *one word* in ordinary language; this is understandable: it makes his proof more transparent. However, it is as immaterial to the proof as Aristotle's invariable custom of choosing triads two of whose members coincide. As far as the proof itself is concerned, two triads which had *no* members in common would suffice.

Aristotle's method of refutation is thus not only "correct" but perfectly adapted to the facts of the case. There are good reasons why Aristotle chooses for his proof of syllogisms a method which can be called axiomatic: some syllogisms satisfy the first requirement of an Aristotelian axiom: *self-evidence*.[76] If in his proof of the inconcludence of pairs Aristotle falls back on giving counter-instances, this answers to the nature of the relation between universal propositions and their negations: the negation of a proposition bound by a universal quantifier must contain an existential quantifier. Philoponus, clearly following Aristotle's lead, was well aware of this: at the end of his discussion of Aristotle's first proof of inconcludence he says: "Ten thousand examples cannot prove a universal proposition, but *one* example is enough to refute it."[77]

NOTES

1. *APst. A* 2, 71b20–23: ἀνάγκη καὶ τὴν ἀποδεικτικὴν ἐπιστήμην ἐξ ἀληθῶν τ' εἶναι καὶ πρώτων καὶ ἀμέσων καὶ γνωριμωτέρων καὶ προτέρων καὶ αἰτίων τοῦ συμπεράσματος.
2. *APst. A* 3, 72b18–22: ἡμεῖς δέ φαμεν οὔτε πᾶσαν ἐπιστήμην ἀποδεικτικὴν εἶναι, ἀλλὰ τὴν τῶν ἀμέσων ἀναπόδεικτον (καὶ τοῦθ' ὅτι ἀναγκαῖον, φανερόν· εἰ γὰρ ἀνάγκη μὲν τὸ ἐπίστασθαι τὰ πρότερα καὶ ἐξ ὧν ἡ ἀπόδειξις, ἵσταται δέ ποτε τὰ ἄμεσα, ταῦτ' ἀναπόδεικτα ἀνάγκη εἶναι).
   The difficulties in b22 (cf. Ross, APPA, p. 514) are removed by H. Schöne's reading: ... ποτε, τὰ ἄμεσα ταῦτ' ... (*Blätter für deutsche Philosophie* **4**, p. 264).
3. *APst. A* 2, 71b17 sq.: ἀπόδειξιν δὲ λέγω συλλογισμὸν ἐπιστημονικόν.
4. *APst. A* 14, 79a31 sq.: φανερὸν οὖν ὅτι κυριώτατον τοῦ ἐπίστασθαι τὸ πρῶτον σχῆμα.
5. ἐκ προγιγνωσκομένων δὲ πᾶσα διδασκαλία, ὥσπερ καὶ ἐν τοῖς ἀναλυτικοῖς λέγομεν. ἡ μὲν γὰρ δι' ἐπαγωγῆς, ἡ δὲ συλλογισμῷ ... εἰσὶν ἄρα ἀρχαὶ ἐξ ὧν ὁ συλλογισμός, ὧν οὐκ ἔστι συλλογισμός· ἐπαγωγὴ ἄρα (*Eth. Nic. Z* 3, 1139b26–31).
6. The question whether syllogistic is the whole or only a part of logic was, notoriously, a major point of conflict between the disciples of modern mathematical logic (logistic) and their opponents. It is interesting to observe that Boole's work, *An Investigation of the Laws of Thought* (1847), which blazed the trail for modern logic, contains an exemplary discussion of this question: pp. 238–241.
7. It is customary to distinguish predicate from propositional logic by asserting that the latter is the more elementary ("more fundamental": Łukasiewicz, AS, p. 47), since it can and does leave undiscussed what Quine (*Methods of Logic*, p. 65) calls "the finer substructures" of the propositions which the statement letters *p*, *q*, *r* ... of its theorems represent. Some logicians are pleased to speak as if logical analysis, starting from multi-propositional complexes, took its first step by analysing these into their constituent propositions (propositional logic) and left for the second step the analysis of simple propositions into their smallest elements, terms (predicate logic). Cf. Quine, l.c. In that case it is hard to see why from a historical point of view the logic of predicates was founded first (by Aristotle) and the logic of propositions was not developed until later (by the Stoics). It must seem as if Aristotle took the second step before the first.

   However, we can only call one thing more elementary than another within some determinate framework. Frege's maxim "Ask for the meaning of a word in the context of a proposition, not in isolation" (*Grundlagen der Arithmetik*, 1884, Introduction, p. X), was in its time revolutionary and arose from his experiences with difficult logical problems; it must have seemed perfectly natural to Aristotle to begin the logical analysis, to which the title of his treatise refers, with the *simplest* elements of propositions – that is, with terms. In this equally legitimate sense *syllogistic* is elementary, since it investigates the logical relations between terms.
8. Thus at *APr. B* 2, 53b12 sqq. Aristotle sets out the law: If proposition *A* entails proposition *B*, then not-*B* entails not-*A*. (εἰ γὰρ τοῦ *A* ὄντος ἀνάγκη τὸ *B* εἶναι, τοῦ *B* μὴ ὄντος ἀνάγκη τὸ *A* μὴ εἶναι) He says explicitly that *A* must be taken as a *propositional* variable: τὸ οὖν *A* ὥσπερ ἓν κεῖται, δύο προτάσεις συλληφθεῖσαι (b23–24).
9. ἀνάγεσθαι: *A* 7, 29b1; 18; *A* 23, 40b19; 41b4. – τελειοῦσθαι or ἐπιτελεῖσθαι: *A* 5, 28a5; *A* 6, 29a16; *A* 7; 29b3; 6; 20; *A* 19, 39a1; *A* 23, 40b18; 41b3. – περαίνεσθαι:

A 14, 33a20; *A* 7, 29a32. For the meaning of "τελειοῦσθαι" cf. *de Gen. An. Δ* 6, 774b6: τῶν ζῳοτόκων τὰ μὲν ἀτελῆ προίεται ζῷα, τὰ δὲ τετελειωμένα; for "περαίνεσθαι", the famous definition of tragedy, *Poet.* 6, 1449b24–28: ἔστιν οὖν τραγῳδία μίμησις πράξεως ... δι' ἐλέου καὶ φόβου περαίνουσα τὴν τῶν τοιούτων παθημάτων κάθαρσιν.

10. This is what Alexander means when he says that συλλογισμοὶ ἀτελεῖς need "unveiling" (τοῦ ἐκκαλυφθῆναι: *in APr.* 24, 10).
11. A syllogism which is transformed into a perfect first figure inference has thereby become a different syllogism. If the statement that *it* is 'perfected' is to make sense, it would have to remain the same through the transformation. Perhaps Aristotle is thinking of arguments with concrete terms. Even after an argument has been changed into a perfect syllogism, it stays recognisably the same argument, since only its form, and not the terms of which it treats, is altered. However, where it is a question of proving the validity of a syllogism, arguments with concrete terms may not be deployed, since an example can prove nothing universal. Nevertheless, it is quite possible that Aristotle had concrete terms in mind when he talked of the 'perfecting' of an argument. I owe this point to Professor Josef König.
12. We might also appeal to Aristotle's inconsistent use of both "δεῖξις" and "ἀπόδειξις" for "reduction". Normally in Aristotle "δεῖξις" is the more general term; it includes both ἐπαγωγή and ἀπόδειξις. Sometimes, however, it stands in opposition to ἀπόδειξις and denotes an immediately effective demonstration, one which convinces without any roundabout argument. Because of these two meanings of "δείκνυσθαι" Aristotle's terminology in the *Analytics* tends to vacillate: thus he says of *Celarent* that δέδεικται πρότερον (*A* 5, 27a8–9.), which cannot mean that it has been 'proved' – since it is *evident*. On the other hand, every proof is a δεῖξις: e.g. Aristotle calls the proof of a proposition by means of a syllogism a δεῖξις (*A* 4, 26b31). Thus "δεῖξις" primarily means the genus which covers the two species ἀπόδειξις and ἐπαγωγή which together form an exhaustive disjunction of the province of δεῖξις (where, since Aristotle equates ἀπόδειξις with συλλογισμός τις, συλλογισμός may take the place of ἀπόδειξις).

However, Aristotle also uses the adjective δεικτικός to characterise those forms of proof which, compared to proofs of some other type, are more natural, or more immediately convincing. Since every ἀπόδειξις belongs as such to the genus δεῖξις, every ἀπόδειξις ought to be a δεικτικὴ ἀπόδειξις. However, Aristotle distinguishes δεικτικαὶ ἀποδείξεις from those ἀποδείξεις which are not δεικτικαί. E.g. he calls the reduction of syllogisms to the moods of the first figure a δεικτικὴ ἀπόδειξις if it is carried out by means of *conversion* – in contrast to reductio ad impossibile, which clearly appeared to him to be more roundabout and not so easy to follow; again, he calls a proof from affirmative propositions a δεικτικὴ ἀπόδειξις (*APst.* A 25) in contrast to one in which negative propositions appear, since a negative premiss can itself only be proved from a negative *and* an affirmative proposition and thus needs for its proof premisses of *different* qualities. And this fact seemed important enough to Aristotle to warrant his distinguishing two types of proof. The meaning of "δεικτικός", when it is used to mark out one type of proof as superior to another, is, I think, inadequately rendered by the English word "direct" (this does not explain how Aristotle could call an affirmative *proposition* δεικτικός in contrast to a negative one): the word "ostensive" seems to me to capture the required meaning fairly exactly.

An analogous vagueness marks the use of the word ἀπόδειξις where it denotes

the reduction of syllogisms to the first figure. In itself ἀπόδειξις is defined as a syllogism from true and 'first', i.e. underivable, premisses. However, in *A* 5 and *A* 6, Aristotle calls the different methods of reduction ἀποδεῖξεις, including both the reductio ad impossible of *Bocardo* to *Barbara* (*A* 5, 27b3) and the conversion of *Darapti* to *Darii*, where he explicitly says that the ἀπόδείξεις could also be carried out by ecthesis and reductio ad impossibile (*A* 6, 28a23); cf. the reductions of *Felapton* (28a28) and *Datisi* (28b14). Either Aristotle is here extending the meaning of "ἀπόδειξις" or else he is implying that the so-called reductions are themselves syllogisms. Our earlier scrutiny of the text makes the latter more probable: Aristotle represents the fact that the second and third figure syllogisms can be proved by reduction to the first figure as if these first figure moods were *themselves* the required proofs; in fact only the *reduction* to these moods can serve as a proof. Łukasiewicz' assertion that "(Aristotle) does not 'demonstrate' or 'prove' the imperfect syllogisms but 'reduces' them ... to the perfect" (AS, p. 44), can only be accepted with important reservations.

13. εἰ δὲ παντὶ τὸ *A* τῷ *B*, καὶ τὸ *B* τινὶ τῷ *A* ὑπάρξει· εἰ γὰρ μηδενί, οὐδὲ τὸ *A* οὐδενὶ τῷ *B* ὑπάρξει· ἀλλ' ὑπέκειτο παντὶ ὑπάρχειν (*A* 2, 25a17–19).
14. The theorem can also be made clear in this way: since p follows from $p \& \sim q$, and thus $(p \& \sim q) \rightarrow (p \& \sim p)$ follows from $(p \& \sim q) \rightarrow \sim p$, a *contradiction* follows from the $(p \& \sim q)$ in $(p \& \sim q) \rightarrow \sim p$. A proposition which entails a contradiction is false: therefore the negation of $p \& \sim q$, $p \rightarrow q$, is true. (I owe this to Prof. P. Lorenzen.)
15. κατηγορείσθω γὰρ τὸ *M* τοῦ μὲν *N* μηδενός, τοῦ δὲ *Ξ* παντός. ἐπεὶ οὖν ἀντιστρέφει τὸ στερητικόν, οὐδενὶ τῷ *M* ὑπάρξει τὸ *N*, τὸ δέ γε *M* παντὶ τῷ *Ξ* ὑπέκειτο· ὥστε τὸ *N* οὐδενὶ τῷ *Ξ*· τοῦτο γὰρ δέδεικται πρότερον (*A* 5, 27a5–9).
16. ὅταν καὶ τὸ *Π* καὶ τὸ *P* παντὶ τῷ *Σ* ὑπάρχῃ, ... τινὶ τῷ *P* τὸ *Π* ὑπάρξει ἐξ ἀνάγκης. ἐπεὶ γὰρ ἀντιστρέφει τὸ κατηγορικόν, ὑπάρξει τὸ *Σ* τινὶ τῷ *P*, ὥστ' ἐπεὶ τῷ μὲν *Σ* παντὶ τὸ *Π*, τῷ δὲ *P* τινὶ τὸ *Σ*, ἀνάγκη τὸ *Π* τινὶ τῷ *P* ὑπάρχειν· γίνεται γὰρ συλλογισμὸς διὰ τοῦ πρώτου σχήματος (*A* 6, 28a18–22).
17. This is the better reading. The variant in A^2, "the *N* belongs to no *X*", gives the conclusion of the syllogism straight away. A 'reduction' of course requires the second reading, since we must first describe what we are to reduce: only in a proof can the demonstrandum stand at the end. It is thus of importance for the question whether or not reduction is and is meant to be a *proof*, to decide between the better attested lectio difficilior, upon which Alexander (78, 25 sqq.) founded his interpretation, and the smoother reading which Waitz preferred.
18. The last sentence can be read in different ways. Ross explains (APPA, p. 308): "i.e. so that Camestres reduces to the same argument as Cesare did in a5–9, i.e. to Celarent"; Alexander takes it to mean that *NeX* can be proved by the same syllogism as *XeN* (97, 19). But a syllogism with a different conclusion is certainly no longer the same syllogism. Hence "συλλογισμός" probably means here, as often, simply "conclusion"; and the sentence asserts that the same *conclusion* can be inferred from the present premisses, *ae*, as was inferred from the premisses *ea* of the second figure which have just been discussed.
19. πάλιν εἰ τὸ *M* τῷ μὲν *N* παντὶ τῷ δὲ *Ξ* μηδενί, οὐδὲ τὸ *Ξ* τῷ *N* οὐδενὶ ὑπάρξει (εἰ γὰρ τὸ *M* οὐδενὶ τῷ *Ξ*, οὐδὲ τὸ *Ξ* οὐδενὶ τῷ *M*. τὸ δέ γε *M* παντὶ τῷ *N* ὑπῆρχεν. τὸ ἄρα *Ξ* οὐδενὶ τῷ *N* ὑπάρξει. γεγένηται γὰρ πάλιν τὸ πρῶτον σχῆμα)· ἐπεὶ δὲ ἀντιστρέφει τὸ στερητικόν, οὐδὲ τὸ *N* οὐδενὶ τῷ *Ξ* ὑπάρξει, ὥστ' ἔσται ὁ αὐτὸς συλλογισμός (*A* 5, 27a9–14).

20. I shall take this opportunity to make some remarks to the reader who is acquainted with traditional syllogistic and the meaning of the mnemonics, like "*Camestres*". The letter *s*, which marks the *conversion* of the proposition whose symbol it follows, has two *different* meanings according to whether it follows the symbol for a premiss or for the conclusion of the syllogism to be reduced. This fact is not often noticed. If *s* occurs in the syllable representing a *premiss*, then it means that this premiss must be converted in order to obtain the corresponding premiss of the perfect argument to which the syllogism is to be 'reduced'. If *s* occurs in the syllable marking the *conclusion*, it means exactly the reverse: that the conclusion of the perfect argument to which the syllogism is to be reduced must be converted in order to get the conclusion of the original syllogism. The same holds, of course, for *p*, which marks the traditional 'conversio per accidens' (conversion according to the second of our rules, $AaB \rightarrow BiA$). This is quite clear from the fact that when *p* marks the conversion of a premiss it follows an *a* (*Darapti*, *Felapton*, *Fesapo*), but when it occurs in a conclusion it follows an *i* (*Bamalip*). The premisses of the perfect syllogism are derived by conversion *from* those of the syllogism to be proved: the conclusion of the syllogism to be proved is 'produced' *from* that of the perfect syllogism, in accordance with the rules of propositional logic on which the procedure is based. If *s* and *p* had the same meaning in both premisses and conclusion, if, that is, it were true that a syllogism is valid if a perfect argument can be produced from its premisses and conclusion in accordance with rules (1)–(3) (and this is sometimes actually said), then e.g. $AiB \& CaB \rightarrow AaC$ would be a valid argument: for we could produce from it $CaB \& BiA \rightarrow CiA$ (*Darii*). But this argument is of course invalid; it would only be valid if the following formula were a logical law:

$$[(pq \rightarrow r) \& (st \rightarrow pq) \& (u \rightarrow r)] \rightarrow (st \rightarrow u)$$

This proposition is false if *u* is false and *p*, *q*, *r*, *s*, and *t* are true; its truth therefore depends on the truth of the individual propositions *p*, *q*, *r*, *s*, *t*, *u*, and it is thus not a law of logic.

21. εἰ γὰρ τὸ μὲν *P* παντὶ τῷ *Σ*, τὸ δὲ *Π* τινί, ἀνάγκη τὸ *Π* τινὶ τῷ *P* ὑπάρχειν. ἐπεὶ γὰρ ἀντιστρέφει τὸ καταφατικόν, ὑπάρξει τὸ *Σ* τινὶ τῷ *Π*, ὥστ' ἐπεὶ τὸ μὲν *P* παντὶ τῷ *Σ*, τὸ δὲ *Σ* τινὶ τῷ *Π*, καὶ τὸ *P* τινὶ τῷ *Π* ὑπάρξει· ὥστε τὸ *Π* τινὶ τῷ *P* (*A* 6, 28b7–11).
22. *Top*. *B* 1, 109a3–6: δείξαντες γὰρ ὅτι παντὶ ὑπάρχει, καὶ ὅτι τινὶ ὑπάρχει δεδειχότες ἐσόμεθα. ὁμοίως δὲ κἂν ὅτι οὐδενὶ ὑπάρχει δείξωμεν, καὶ ὅτι οὐ παντὶ ὑπάρχει δεδειχότες ἐσόμεθα.
23. Bocheński (FL, p. 83; HFL, p. 71) takes *APr*. *B* 1, 53a15–30 as a recognition of the subaltern moods, but the passage cannot witness what Bocheński thinks it does. There Aristotle states only that in arguments with *universal* and true conclusions any term subordinate to the minor term may replace it in the conclusion without changing the truth value of the conclusion, and that this is not possible in arguments with particular conclusions. For example, if "Animal belongs to all men" is the conclusion of a syllogism with true premisses, then "Animal belongs to all Greeks" is also a true proposition. On the other hand, if "Man belongs to some animals" is the conclusion of a syllogism with true premisses, then, although "lion" is a subordinate term to "animal", "Man belongs to some lions" is not true. We have already come across Apuleius' testimony that the subaltern moods were first introduced by Ariston (c. 50 BC), whereas all the moods which could

be derived from Aristotle's general declarations in *A* 7 and *B* 1 had already been introduced into the system by Theophrastus. It would be amazing if it had taken the Peripatetic logicians 250 years to take up the hints of this passage. (Cf. p. 111).

24. Łukasiewicz' discussion of the 'direct' methods of reduction (AS, § 17, pp. 51–54) is, like so much in his book, uncommonly acute and instructive – in particular his rigorous formalisation of Aristotle's arguments. However, Łukasiewicz seems here to stray pretty far from his purpose, the interpretation of the text of the *Analytics*. Admittedly, nothing can be done without the use of propositional laws; but it is illegitimately anachronistic to ascribe to Aristotle 'intuitions' into these operations – which is what Łukasiewicz does ("analysing his intuitions" AS, p. 51).
25. πάλιν εἰ τῷ μὲν *N* παντὶ τὸ *M*, τῷ δὲ *Ξ* τινὶ μὴ ὑπάρχει, ἀνάγκη τὸ *N* τινὶ τῷ *Ξ* μὴ ὑπάρχειν· εἰ γὰρ παντὶ ὑπάρχει, κατηγορεῖται δὲ καὶ τὸ *M* παντὸς τοῦ *N*, ἀνάγκη τὸ *M* παντὶ τῷ *Ξ* ὑπάρχειν. ὑπέκειτο δὲ τινὶ μὴ ὑπάρχειν (*A* 5, 27a36–b1).
26. APPA, p. 319: Aristotle "assumes (in a reductio ad impossibile) the falsity of an original conclusion in order to prove its validity (sic)"; cf. p. 31: "To validate the inference involved in Baroco .... Therefore that some *S* is not *P* must be true".
27. AS, p. 54: "Usually it is explained in the following way."
28. Here again we meet with an idiom which shunts our ideas down a side-line and marshals them towards the wrong destination: the fact that reduction proves the validity of a *syllogism* is obscurely expressed by the misleading statement that reduction shows the conclusion of the syllogism to be its actual conclusion – it is but a short step from here to the notion that reduction proves the *proposition* which is the conclusion of the reduced syllogism. Thus Alexander says in his discussion of the last step in the proof of *Camestres*: καὶ οὕτως δείκνυσι τὸ *N* μηδενὶ τῷ *Ξ* ὑπάρχον, ὃ ἔδει δειχθῆναι συναγόμενον (79, 13–14). Had he said simply ὃ ἔδει δειχθῆναι, we should have had to attribute the traditional error to him. However, his text reads: "which was to be proved as *conclusion*", and this may be interpreted both as the correct "which was to be proved to be the conclusion of a syllogism", and as the incorrect "which conclusion was to be proved". This is exactly similar to the case discussed on p. 24: we showed that the mistaken reading of syllogistic necessity as an operator on the *proposition* which occurs as conclusion must be encouraged by Aristotle's habit of speaking of the relative necessity of the conclusion *as such*. Similar 'systematically misleading expressions' can be traced at the roots of almost all the traditional misunderstandings which we discuss in this book. This fact has, I think, a symptomatic significance for philosophy and its history: this is the way in which the most obstinate errors grow up. (The phrase "systematically misleading expression" comes from Gilbert Ryle's paper of that title, in *Logic and Language*, ed. Anthony Flew, series I, 1951, pp. 11–36.)
29. Cf. above, n. 25.
30. On this cf. Ernst Kapp, *Greek Foundations of Traditional Logic*, New York, 1942, p. 19.
31. *APr. A* 23, 41a23–40; *A* 29, 45a23–b20; *A* 44, 50a15–b4; *B* 11, passim; *APst. A* 11, 77a22–25.
32. *A* 44, 50a32–38: διαφέρουσι δὲ (sc. οἱ διὰ τοῦ ἀδυνάτου συλλογισμοὶ) τῶν προειρημένων (τῶν ἄλλων τῶν ἐξ ὑποθέσεως) ὅτι ἐν ἐκείνοις μὲν δεῖ προδιομολογήσασθαι, εἰ μέλλει συμφήσειν ... ἐνταῦθα δὲ καὶ μὴ προδιομολογησάμενοι συγχωροῦσι διὰ τὸ φανερὸν εἶναι τὸ ψεῦδος. Cf. Ross, APPA, pp. 31 and 417.
33. *A* 44, 50a16–17: ἔτι δὲ τοὺς ἐξ ὑποθέσεως συλλογισμοὺς οὐ πειρατέον ἀνάγειν.

29–39: ὁμοίως δὲ καὶ ἐπὶ τῶν διὰ τοῦ ἀδυνάτου περαινομένων· οὐδὲ γὰρ τούτους οὐκ ἔστιν ἀναλύειν.

34. *A* 29, 45a26–28: ὃ γὰρ δείκνυται δεικτικῶς, καὶ διὰ τοῦ ἀδυνάτου ἔστι συλλογίσασθαι διὰ τῶν αὐτῶν ὅρων· καὶ ὃ διὰ τοῦ ἀδυνάτου, καὶ δεικτικῶς; cf. also *B* 14, 62b38–40.
35. False because it leads to a *contradiction*: from the supposition that both premisses are true and the conclusion is false, it *follows* that one of the premisses is false. Therefore our opponent must affirm at the same time that both premisses are true and that one premiss is false, i.e. that not both premisses are true. This is the eponymous impossibility of reductio ad impossibile.
36. *APr. B* 2, 53b12–13; *B* 4, 57b1–2. Cf. above, 184, n. 8.
37. *APr. B* 8, 59b3–5: ἀνάγκη γὰρ τοῦ συμπεράσματος ἀντιστραφέντος καὶ τῆς ἑτέρας μενούσης προτάσεως σναιρεῖσθαι τὴν λοιπήν· εἰ γὰρ ἔσται, καὶ τὸ συμπέρασμα ἔσται.
38. Leibniz, *De formis syllogismorum mathematice definiendis*, (*Opuscules et fragments inédits*, ed. L. Couturat, Paris, 1903, reprinted Hildesheim, 1961, pp. 413–414; *Leibniz: Logical Papers, a Selection*, ed. G.H.R. Parkinson, Oxford, 1966, pp. 105–111), cf. Bocheński, FL, pp. 303–304; HFL, pp. 259–260.
39. J.N. Keynes, *Formal Logic*[4], London, 1906, pp. 333-334.
40. A. Schopenhauer, *Über Schriftstellerei und Stil*, *Werke*, ed. P. Deussen, vol. 5, p. 565.
41. τὸ δὲ ἅπαν φάναι ἢ ἀποφάναι ἡ εἰς τὸ ἀδύνατον ἀπόδειξις λαμβάνει. Cf. too *APr. B* 11, 62a12–15: φανερὸν οὖν ὅτι ... τὸ ἀντικείμενον ὑποθετέον ἐν ἅπασι τοῖς συλλογισμοῖς (sc. διὰ τοῦ ἀδυνάτου). εἰ γὰρ κατὰ παντὸς ἡ φάσις ἢ ἡ ἀπόφασις, δειχθέντος ὅτι οὐχ ἡ ἀπόφασις, ἀνάγκη τὴν κατάφασιν ἀληθεύεσθαι.
42. Cf. *Cat.* 12, 14b15–18: εἰ γὰρ ἔστιν ἄνθρωπος, ἀληθὴς ὁ λογος ᾧ λέγομεν ὅτι ἔστιν ἄνθρωπος· καὶ ἀντιστρέφει γε, – εἰ γὰρ ἀληθὴς ὁ λόγος ᾧ λέγομεν ὅτι ἔστιν ἄνθρωπος, ἔστιν ἄνθρωπος.
43. *APr. A* 6, 28a22–26: ἔστι δὲ καὶ διὰ τοῦ ἀδυνάτου καὶ τῷ ἐκθέσθαι ποιεῖν τὴν ἀπόδειξιν. εἰ γαρ ἄμφω παντὶ τῷ *Σ* ὑπάρχει, ἂν ληφθῇ τι τΠν *Σ* οἷον τὸ *N*, τούτῳ καὶ τὸ *Π* καὶ τὸ *P* ὑπάρξει, ὥστε τινὶ τῷ *P* τὸ Π ὑπάρξει.
44. *in APr.* 99, 28–32: τί γὰρ διαφέρει τῷ *Σ* ὑπάρξειν λαβεῖν παντὶ τό τε *Π* καὶ *P* καὶ μέρει τινὶ τοῦ *Σ* τῷ *N*; τὸ γὰρ αὐτὸ καὶ ἐπὶ τοῦ *N* ληφθέντος μένει· ἡ γὰρ αὐτὴ συζυγία ἐστιν, ἄν τε κατὰ τοῦ *N* παντὸς ἐκείνων ἑκάτερον, ἄν τε κατὰ τοῦ *Σ* κατηγορεῖται. ἢ οὐ τοιαύτη ἡ δεῖξις ᾗ χρῆται· ὁ γὰρ δι' ἐκθέσεως τρόπος δι' αἰσθήσεως γίνεται.
45. $\exists$ is the symbol used by Peano and adopted by Whitehead and Russell (PM I, p. 127) for the existential quantifier. We have already, p. 53, used this operator in connexion with individual variables; here it governs a term variable. "$\exists$" should be read "there is"; thus "$(\exists C)(AaC)$" reads "There is a term C such that A belongs to all C".
46. *A* 28, 43b43–44a2: ἢν δὲ μὴ ὅτι παντὶ ἀλλ' ὅτι τινί (sc. ὑπάρχει βουλώμεθα κατασκευάζειν, βλεπτέον εἰς ταῦτα) οἷς ἕπεται ἑκάτερον· εἰ γάρ τι τούτων ταὐτόν, ἀνάγκη τινὶ ὑπάρχειν.
47. This formulation would lead, not to (*o*), but to: $AoB \rightarrow (\exists C)(CeA \,\&\, BaC)$. But $CeA \,\&\, BaC$ are the premisses of *Fesapo* (IV) – which Aristotle does not officially recognise: cf. § 25 – and yield AoB. Ross (APPA, p. 387–388) notes this and allows Aristotle to convert CeA "at once" to AeC, so that the premisses of *Felapton* – and also our formula (*o*) – result.

48. *A* 28, 44a9–11: ἐὰν δὲ τινὶ μὴ ὑπάρχειν (sc. δέῃ κατασκευάζειν, βλεπτέον) ᾧ μὲν δεῖ μὴ ὑπάρχειν, οἷς ἕπεται, ὃ δὲ μὴ ὑπάρχειν, ἃ μὴ δύναται αὐτῷ ὑπάρχειν· εἰ γάρ τι τούτων εἴη ταὐτόν, ἀνάγκη τινὶ μὴ ὑπάρχειν.
49. *A* 2, 25a15–17: εἰ οὖν μηδενὶ τῷ *B* τὸ *A* ὑπάρχει, οὐδὲ τῷ *A* οὐδενὶ ὑπάρξει τὸ *B*· εἰ γὰρ τινι οἷον τῷ *Γ*, οὐκ ἀληθὲς ἔσται τὸ μηδενὶ τῷ *B* τὸ *A* ὑπάρχειν· τὸ γὰρ *Γ* τῶν *B* τί ἐστιν.
50. Waitz puts it neatly: "Aristoteles hoc non tam demonstravit, quam quomodo demonstrari possit innuit" (I, p. 374). However, in his explanation of this hint Waitz follows Alexander's view that it must be an *individual* which is 'exposed'.
51. *in APr.* 33, 15: οὐδέπω γὰρ περὶ τῶν συλλογιστικῶν δείξεων γνώριμον.
52. ib. 33, 13: τῇ δι' ἐκθέσεως δείξει οὕτως κέχρηται ὡς οὔσῃ αἰσθητικῇ ἀλλ' οὐ συλλογιστικῇ.
53. *A* 6, 28b20–21: δείκνυται δὲ καὶ ἄνευ τῆς ἀπαγωγῆς, ἐὰν ληφθῇ τι τῶν *Σ* ᾧ τὸ *Π* μὴ ὑπάρχει.
54. *in APr.* 274, 19–24: δι' ὧν δὲ λέγει νῦν, ὑπογράφει ἡμῖν φανερώτερον τὸ λεγόμενον συνθετικὸν θεώρημα, οὗ αὐτός ἐστιν εὑρετής. ἔστι δὲ ἡ περιοχὴ αὐτοῦ τοιαύτη· ὅταν ἔκ τινων συνάγηταί τι, τὸ δὲ συναγόμενον μετά τινος ἢ τινῶν συνάγῃ τι, καὶ τὰ συνακτικὰ αὐτοῦ, μεθ' οὗ ἢ μεθ' ὧν συνάγεται ἐκεῖνο, καὶ αὐτὰ τὸ αὐτὸ συνάξει.

    If, e.g. the conclusion of a syllogism *C* and another proposition *D* together entail *E*, then *E* also follows from the premisses of *C* together with *D*. It is called a synthetic theorem because it can take a sequence of syllogisms and, by suppressing certain of their conclusions, form a *new* syllogism from them. Alexander is here commenting on *A* 25, the chapter on polysyllogisms. The synthetic theorem, put in terms of propositional logic, would look like this:

$$([(p\,\&\,q)\rightarrow r]\,\&\,[(r\,\&\,s)\rightarrow t])\rightarrow[(p\,\&\,q\,\&\,s)\rightarrow t]$$

55. Whitehead and Russell, PM, I, p. 112, 3.3:

$$[(p\,\&\,q)\rightarrow r]\rightarrow[p\rightarrow(q\rightarrow r)] \quad \text{('exportation').}$$

    i.e.: "If p and q together imply r, then p alone implies that q implies r".
56. This law is the converse to that stated in the preceding note. Cf. PM, I, p. 112, 3.31:

$$[p\rightarrow(q\rightarrow r)]\rightarrow[(p\,\&\,q)\rightarrow r] \quad \text{('importation').}$$

57. *A* 8, 30a6–14: ἐν μὲν οὖν τοῖς ἄλλοις τὸν αὐτὸν τρόπον δειχθήσεται διὰ τῆς ἀντιστροφῆς τὸ συμπέρασμα ἀναγκαῖον, ὥσπερ ἐπὶ τοῦ ὑπάρχειν. ἐν δὲ τῷ μέσῳ σχήματι, ὅταν ᾖ τὸ καθόλου καταφατικὸν τὸ δ' ἐν μέρει στερητικόν (*Baroco*), καὶ πάλιν ἐν τῷ τρίτῳ, ὅταν τὸ μὲν καθόλου κατηγορικὸν τὸ δ' ἐν μέρει στερητικόν (*Bocardo*), οὐχ ὁμοίως ἔσται ἡ ἀπόδειξις, ἀλλ' ἀνάγκη **ἐκθεμένους** ᾧ τινὶ ἑκάτερον μὴ ὑπάρχει, κατὰ τούτου ποιεῖν τὸν συλλογισμόν· ἔσται γὰρ ἀναγκαῖος ἐπὶ τούτων· εἰ δὲ κατὰ τοῦ ἐκτεθέντος ἐστὶν ἀναγκαῖος, καὶ κατ' ἐκείνου τινός. τὸ γὰρ ἐκτεθὲν ὅπερ ἐκεῖνό τί ἐστιν. **γίνεται δὲ τῶν συλλογισμῶν ἑκάτερος ἐν τῷ οἰκείῳ σχήματι.**
58. I shall disregard the modal operators in what follows for the sake of clarity.
59. For a discussion of W. Albrecht's *Die Logik der Logistik*, and the interpretation of ecthesis offered there, cf. the German edition, pp. 178–180.
60. This is false: *CoA* follows from *AaB*&*BeC*. Cf. pp. 55–56.
61. *A* 4, 26a2–9: (a) εἰ δὲ τὸ μὲν πρῶτον παντὶ τῷ μέσῳ ἀκολουθεῖ, τὸ δὲ μέσον

μηδενὶ τῷ ἐσχάτῳ ὑπάρχει, οὐκ ἔσται συλλογισμὸς τῶν ἄκρων. (b) οὐδὲν γὰρ ἀναγκαῖον συμβαίνει τῷ ταῦτα εἶναι. (c) καὶ γὰρ παντὶ καὶ μηδενὶ ἐνδέχεται τὸ πρῶτον τῷ ἐσχάτῳ ὑπάρχειν, ὥστε οὔτε τό κατὰ μέρος οὔτε τὸ καθόλου γίνεται ἀναγκαῖον. (d) μηδενὸς δὲ ὄντος ἀναγκαίου διὰ τούτων οὐκ ἔσται συλλογισμός. (e) ὅροι τοῦ παντὶ ὑπάρχειν ζῷον – ἄνθρωπος – ἵππος τοῦ μηδενὶ ζῷον – ἄνθρωπος – λίθος.

62. *in APr.* 55, 21–26: ὅτι γὰρ οὕτως ἐχούσων τῶν προτάσεων οὐδὲν ἀναγκαῖον συνάγεται (a *bad* paraphrase of Aristotle's "ἀναγκαῖον συμβαίνει") ὅ ἐστιν ἴδιον συλλογισμοῦ, αὐτὸς μὲν δείκνυσι τῇ τῆς ὕλης παραθέσει. καὶ γὰρ καθόλου καταφατικὸν ἐπί τινος ὕλης δείξει δυνάμενον συνάγεσθαι καὶ πάλιν ἐπ' ἄλλης καθόλου ἀποφατικόν, ὃ ἐναργέστατον σημεῖον τοῦ μηδεμίαν ἔχειν τὴν συζυγίαν ταύτην ἰσχὺν συλλογιστικήν, εἴ γε τὰ ἐναντία καὶ ἀντικείμενα ἐν αὐτῇ δείκνυται, ὄντα ἀλλήλων ἀναιρετικά.
63. E. Rolfes, *Aristoteles' Lehre vom Schluss*, Phil. Bibl., Band 10, Leipzig, 1921, (ed.[2] 1948), p. 150, n. 7.
64. Ross is right to add this, since in the first figure *CoA* follows from *ae*. We have already discussed Aristotle's mistake in speaking as if nothing followed at all (p. 55).
65. Bocheński's brief discussion of Aristotle's method of rejection (FL, p. 77; HFL, p. 66) is incorrect. The method expounded by Lorenzen (*Formale Logik*, Sammlung Göschen, Band 1176/76a, 1958, pp. 27 sqq.) is the genuine Aristotle.
66. Peano, the Italian mathematician (1858–1932), introduced "ε" to symbolise the relation which an individual bears to every class to which it belongs. "$p \varepsilon P$" should be read "*p* belongs to the class *P*" or "*p* is one of the *P*'s".
67. A. de Morgan, *Formal Logic*, London, 1847. The laws were already familiar to Ockham and Burleigh: cf. Bocheński, FL, p. 241; HFL, p. 207.
68. Viz "Animal (*A*), man (*B*), horse (*C*)" and "Animal (*A*), man (*B*), stone (*C*)".
69. E.g. the triples "Vertebrate, man, mammal" and "vertebrate, man, animal" ("man" is the middle term).
70. The Greek text reads ἀκολουθήσει instead of ἐνδέχεται ἀκολουθεῖν. We have discussed the mistake above, p. 41, n. 10.
71. *A* 4, 26a39–b10: οὐδ' ὅταν τὸ μὲν πρὸς τῷ μείζονι ἄκρῳ καθόλου γένηται ἢ κατηγορικὸν ἢ στερητικόν, τὸ δὲ πρὸς τῷ ἐλάττονι στερητικὸν κατὰ μέρος, οὐκ ἔσται συλλογισμός, οἷον εἰ τὸ μὲν *A* παντὶ τῷ *B* ὑπάρχει, τὸ δὲ *B* τινὶ τῷ *Γ* μή, ἢ εἰ μὴ παντὶ ὑπάρχει· ᾧ γὰρ ἄν τινι μὴ ὑπάρχῃ τὸ μέσον, τούτῳ καὶ παντὶ καὶ οὐδενὶ ἀκολουθήσει τὸ πρῶτον. ὑποκείσθωσαν γὰρ οἱ ὅροι ζῷον, ἄνθρωπος, λευκόν, εἶτα καὶ ὧν μὴ κατηγορεῖται λευκῶν ὁ ἄνθρωπος, εἰλήφθω κύκνος καὶ χιών· οὐκοῦν τὸ ζῷον τοῦ μὲν παντὸς κατηγορεῖται, τοῦ δὲ οὐδενός, ὥστε οὐκ ἔσται συλλογισμός.
72. Since the argument is supposed to prove the assertion of 26b5–6: "*A* can belong to all or to none of the term (*C*) to some of which *B* does not belong."
73. *A* 4, 26b14–20: ἔτι ἐπεὶ ἀδιόριστον τὸ τινὶ τῷ *Γ* τὸ *B* μὴ ὑπάρχειν, ἀληθεύεται δὲ καὶ εἰ μηδενὶ ὑπάρχει καὶ εἰ μὴ παντὶ ὅτι τινὶ οὐχ ὑπάρχει, ληφθέντων δὲ τοιούτων ὅρων ὥστε μηδενὶ ὑπάρχειν οὐ γίνεται συλλογισμός (τοῦτο γὰρ εἴρηται πρότερον), φανερὸν οὖν ὅτι τῷ οὕτως ἔχειν τοὺς ὅρους οὐκ ἔσται συλλογισμός· ἦν γὰρ καὶ ἐπὶ τούτων.
74. In symbols:

$$([Not\colon (p\,\&\,q \rightarrow r)]\,\&\,(q \rightarrow s)) \rightarrow [Not\colon (p\,\&\,s \rightarrow r)]$$

75. *A* 5, 27b16–21: τοῦ δὲ παντὶ ὑπάρχειν οὐκ ἔστι λαβεῖν, εἰ τὸ *M* τῷ *Ξ* τινὶ μὲν

ὑπάρχει τινὶ δὲ μή. εἰ γὰρ παντὶ τῷ *Ξ* τὸ *N*, τὸ δὲ *M* μηδενὶ τῷ *N*, τὸ *M* οὐδενὶ τῷ *Ξ* ὑπάρξει. ἀλλ' ὑπέκειτο τινὶ ὑπάρχειν. οὕτω μὲν οὖν οὐκ ἐγχωρεῖ λαβεῖν ὅρους, ἐκ δὲ τοῦ ἀδιορίστου δεικτέον.

76. *APst. A* 9, 76b23–24: οὐκ ἔστι δ' ὑπόθεσις οὐδ' αἴτημα (sc. ἀλλ' ἀξίωμα), ὃ ἀνάγκη εἶναι δι' αὐτὸ καὶ **δοκεῖν** ἀνάγκη (cf. Ross, APPA, p. 540).
77. *In APr.* 75, 14–15: τὸν μὲν γὰρ καθόλου λόγον οὐδὲ μυρία παραδείγματα ἱκανὰ κατασκευάσαι, ἀνασκευάσαι δὲ καὶ ἓν ἐξαρκεῖ παράδειγμα.

CONCLUSION

We have trailed across Aristotle's syllogistic from five different starting-points. Our expeditions were concerned with the resolution of particular problems: we can now speak in more general terms on the basis of our observations. We have asked What is an Aristotelian syllogism? What a perfect syllogism? And what a figure in Aristotle's sense? We have also investigated the logical aids Aristotle uses to make his syllogistic into a system. There remains the final question: What is Aristotle's syllogistic?

Characteristically, Aristotle himself never asks the question. Indeed, he seems positively to avoid it: his terminology is wonderfully neutral; the statements which prepare the way for logical argument proper are deliberately couched in lax and loose formulations; when the basic questions about the real nature of the formal system must impress themselves on every reader, Aristotle is reserved and non-committal. When we put our final question to the text we are faced with particular difficulty.

First of all: the *Prior Analytics* is a treatise not on methodology but on the *theory* of inference. It investigates the relations between terms (ὅροι) which can function as values of the variables in propositions of the form "*A* belongs/does not belong to all/no/some *B*". It is not unaristotelian to consider the logical constants *a*, *e*, *i*, and *o* as *relations* between terms, as we can see from the fact that he sometimes replaces them by the expression "πρός" which is his standard word for relations (e.g. *APr. A* 7, 29a27). Thus Aristotle's syllogistic is the theory of the relative products of these binary relations between terms. The cases in which the relative product is itself one of the relations *a*, *e*, *i*, *o*, are designated as syllogisms. Procedures are established whereby it can be proved that a given relative product has one of these values; and a method is given whereby it can be shown that a given relative product has no definite value, that is, has as its value the alternation of *a*, *e*, *i*, and *o*. Since one of these relations *must* always hold between any pair of terms whatever, their alternation holds for every pair of terms and therefore tells us nothing about the logical

relation between them. Aristotle's theory is thus a special part of the logic of binary relations.

Such a description of Aristotle's syllogistic will arouse immediate opposition: how can it be reconciled with the protagonistic role which the theory has played in the history of logic and philosophy? Not long ago indeed it was equated with logic itself. The answer is this: a special theory cannot be universal, but it can have a universal *significance*; and it is clear that a theory which teaches us to survey the logical connexions between propositions of the form AxB may have a wide significance. Besides, it is in a certain sense fundamental, since it investigates the logical relations between the *elements* of propositions – terms. There is, therefore, no reason for amazement that the founder of logic first treated systematically the theory of the syllogism. And, thirdly, Aristotle fulfilled his self-imposed task in such an exemplary fashion that his syllogistic can justly be taken as the paradigm of logical investigation. (This does not exclude the possibility that Aristotle made mistakes; but it entails his having written in such a way that he can be proved to have made them.)

Aristotle's syllogistic is certainly not a 'philosophical' logic; it is not the case that certain philosophical insights are necessary before it can be understood, or that the truth of its statements depends on fixed assumptions which must be accounted ontological or metaphysical. This interpretation of his syllogistic, as we have shown, was prevalent in the nineteenth century, particularly in Germany; we have given sufficient proof of the errors of interpretation to which it has led.

However, it would be wrong to condemn the 'philosophical' explanation of Aristotle's logic as a mere fashion or an ignorant blunder. For those scholars must have seen that the antipathy nurtured by the Enlightenment, by the rise of modern science and by the claims of 'pragmatism' against fruitless and sophistical subtleties and the pedantry of the Schools, was quite misplaced when it was turned against Aristotle, whose intellectual power imparts greatness to every page of the *Analytics*. Thus they gladly abandoned traditional logic to the general contempt, and maintained that all this had nothing to do with Aristotle: for traditional logic was only a degenerate sport of his 'philosophical', metaphysical, logic. This defence of Aristotle was the worst conceivable: if what its protectors said were true, Aristotle's logic would be nothing but an historical curiosity.

The significance of Aristotle's logic lies neither in its universality (for it is a special theory), nor in its philosophical profundity (for this it sedulously avoids), nor yet in its applicability to concrete problems (for Aristotle himself makes very little use of it): its significance lies in its exemplary rigour and in its logical purity.

ARISTOTLE AND SYLLOGISMS FROM FALSE PREMISSES

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In spite of Łukasiewicz's great book on the subject it is still far from being settled if it makes, after all, even sense to discuss Aristotle's syllogistic "from the standpoint of modern formal logic". One of the moot points, as is now well known, is the question whether Aristotle's syllogisms are rules of inference or logical theorems. Traditional logic formulated the syllogisms as rules: *MaP*, *SaM*, *therefore SaP*. Russell and Whitehead (*PM* I, p. 28) have called this a "slipshod way of speaking" as it gives the impression of both premisses and conclusion being *asserted*. Textbooks of logic do not usually say what happens if the premisses in question are false. It might puzzle us for a moment to notice that none other than G. Frege, in a letter to Jourdain from 1910, has put forward the view that "aus falschen Prämissen kann nichts geschlossen werden". But, with Frege, "schliessen" is *defined* as the application of his "Abtrennungsregel", and therefore one cannot really say he would endorse the traditional view.

Now Łukasiewicz has tried to show that the syllogisms of Aristotle are not, like those of traditional logic, *rules* of inference, but genuine *theorems* of logic. His strongest argument is that Aristotle in his systematic discussion of syllogisms and figures (*APr. A* 4–7) introduces his syllogisms in the form of conditional sentences with both premisses as antecedent and the conclusion as consequent, e.g. "If *A* is predicated of all *B* and *B* of all *C*, then it is necessary that *A* should be predicated of all *C*" (*A* 4, 25b37). This thesis of Łukasiewicz's has been contested on different grounds by various scholars. It is called a "strange contention" by Professor Prior.[1] According to him, Aristotle does not, in these chapters, *formulate* syllogisms – he just *talks about* them, and if this is so, his way of talking would be "perfectly natural". It certainly would not imply any differences between the Aristotelian and the traditional syllogism. A German writer, W. Albrecht, takes another line of approach. Łukasiewicz's thesis cannot be true, he says, because Aristotle never recognised syllogisms from false premisses. Now, since a theorem of the form: "*If*

MaP and *SaM*, *then SaP*" would hold even for those terms that do *not* make the premisses true, such a theorem cannot be identified with the Aristotelian syllogism.[2] For his opinion that Aristotle did not think syllogisms with false premisses valid, Albrecht could have claimed the support of H. Maier, who (in his *Syllogistik des Aristoteles* II, 2, p. 246) asserted that in syllogisms with one or two false premisses, according to Aristotle, "the conclusion does not at all issue from the premisses with syllogistic necessity". Maier also has a predecessor: he follows Th. Waitz, who summarised the same passage which Maier had before him in his commentary of 1844 like this: "Explicatur cur conclusio vera, quamquam fieri possit e falsis propositionibus, tamen non necessario ex iis proveniat." Albrecht's view, we must concede, can boast of being founded on some well-established interpretation of a crucial passage of the text of the *Organon*.

I shall for the rest of this paper not comment on Professor Prior's interesting remarks, since they are best discussed along with some other questions he raises in his review of Łukasiewicz's book. For the present I shall concentrate on what might be called the Albrecht argument. I shall try to show by an analysis of the relevant passages (*APr*. *B* 2, 53b4–10 and *B* 4, 57a36–b17) that Aristotle's actual teaching is squarely opposed to the Albrecht, Maier and Waitz interpretation. The analysis of these passages seems interesting also on other grounds: it gives a fair idea of the technical skill of Aristotle's logical discussions and, at the same time, we shall find that Aristotle has a fallacy in his main argument. That fallacy has already been detected by Łukasiewicz[3] and it seems worth while, since Sir David Ross's otherwise excellent commentary is not of much help here, to present the argument in its proper context.

Aristotle starts (53b3–10) with the remark that both premisses of a given syllogism may be true propositions, but that they might as well be both false or of differing truth value. The conclusion is, he says, necessarily either true or false. Now it is impossible to infer a false conclusion from true premisses; but there may be cases in which we can infer a true conclusion from false premisses. But, he adds, such a syllogism can only be a syllogism of the fact, not one of the reason. This he proposes to deal with somewhat later on (ἐν τοῖς ἑπομένοις, 53b10). The reference is here, as in *APr*. *A* 1, 24b14, definitely to *APst*. *A* 2, because only in that chapter Aristotle points out that demonstrative syllogisms which state (in the

premisses) the reason of the fact asserted by the conclusion, *must* have *true* premisses. Maier and Ross, who both think the reference is to the second of our passages (57a36–b17), have mainly on account of this initial mistake precluded themselves from understanding Aristotle's argument.

Aristotle continues (53b11–15) with some reflections that are designed to show that from true premisses only true conclusions can follow. If, he says, from the proposition *A* there follows a proposition *B*, then also Not-*A* follows from Not-*B*. (This is Theorem 11.61 in Bocheński's *Ancient Formal Logic*, Amsterdam, 1951.) Since Aristotle has stated shortly before (*APr. A* 46, 52a32 sqq.) the equivalence between the statement of a fact and the statement that the sentence expressing the fact is true, he is justified in inferring the proposition that, if *A* implies *B*, the truth of *A* implies the truth of *B*. Now the rest is plain sailing: for *A* in this formula Aristotle substitutes the two premisses of a syllogism (53b23; the same substitution we find at *APr. A* 15, 34a22), for *B* he substitutes the conclusion. The truth of the conclusion follows logically, then, from the truth of the premisses. For if we suppose *B* to be not true, then, according to our formula, we could infer the falsity of *A*, the conjunction of the premisses.

It is, however, possible to infer true conclusions from false premisses, but not in all the syllogistic moods. These are now examined by Aristotle in this respect: according to "Barbara", e.g., we can infer the true proposition "All men are animals" from the false premisses, "All men are stones, All stones are animals". After these not very interesting detailed investigations Aristotle continues his general discussion in the second of the passages we want to analyse (57a36–b17). It is evident, he says, that at least one premiss of a correct syllogism must be false if the conclusion is to be false. On the other hand it is not, as one might be tempted to think, necessary that at least one premiss of a syllogism with a true conclusion be *true*. It is, actually, quite possible that the conclusion is true even when both the premisses are false – οὐ μὴν ἐξ ἀνάγκης, but there is no necessity for this. No necessity for what? Obviously for the conclusion being a *true* sentence, not for the conclusion *following* from the premisses, as Maier understands this all-important clause. The whole passage does not deal with the question whether the conclusion follows in such cases, still less with the problem whether the conclusion "follows by necessity" or "simply follows" – which is what Maier actually thinks

Aristotle says (l.c., p. 246). A conclusion is, for Aristotle, *defined* as a proposition that follows from certain premisses with necessity: τὸ μὲν ... γὰρ συμβαῖνον ἐξ ἀνάγκης τὸ συμπέρασμά ἐστιν (53b18).

It is, then, quite off the mark to say, as Maier does, that Aristotle failed, in this passage, to distinguish properly between the truth (*i.e.* the validity) of a syllogism and the truth of the propositions which constitute it. Rather it is Maier who has failed to notice a distinction which Aristotle actually emphasises: Aristotle does not say a true conclusion could not follow with necessity from false premisses, but he says and teaches, consistently, that a conclusion, which follows from false premisses, is not necessarily *true*. That these are two different propositions is clear from the fact that one of them is false, the other true. It is, of course, an overstatement to say that Maier did not see the difference between both these propositions, since he never even thought our second proposition a possible interpretation of the text in question. But Ross regards them, clearly, as equivalent. He says: "The main thesis is that in such a case" (false premisses) "the conclusion does not follow by syllogistic necessity (a40). This is, of course, an ambiguous statement. It might mean that the truth of the conclusion does not follow by syllogistic necessity, but if Aristotle meant this, he would be completely contradicting himself. What he means is that in such a case the premisses cannot state the ground on which the fact stated in the conclusion actually rests, since the same fact cannot be the necessary consequence of another fact and of the opposite of the other."[4] Ross asserts, consequently, that Aristotle shows in 57a40–b3 that the conclusion actually follows, and in b4–17, that the premisses cannot here state the ground on which the fact asserted by the conclusion rests. But we do not find any demonstration of this kind whatever in the text: that a conclusion "actually follows" – if it be a conclusion – is for Aristotle a tautology and does not need any demonstration. And that false premisses do not provide a reason for their true conclusion is certainly an opinion of Aristotle's, but one that has left not even a trace in b4–17. Ross has simply been misled by Aristotle's remark in 53b7 that he would discuss that point later on, as I said before. I confess to being unable to understand the "since" in the quoted passage of Ross' commentary: I do not see how the proposition that one fact cannot be the necessary consequence of another fact and the opposite of this other fact could ever be a *proof* of the proposition

that false premisses cannot give the reason for the fact stated in the conclusion. For there is certainly no necessity for the false premisses in question to be the *negations* of those premisses which contain, in Aristotle's sense, the "ground" of the conclusion.

Aristotle does not, in these passages, discuss the question whether in a syllogism with false premisses the conclusion follows with necessity. Still less does he decide that it does not, as Maier would have us think. Neither does he deal with the problem whether false premisses can contain the ground of the fact asserted by the conclusion, as Ross supposes. The problem Aristotle actually talks about is this: how is it that, while from two true premisses invariably a true conclusion follows and, if the conclusion is false, one of the premisses *must* be false, a conclusion from two false premisses or from premisses differing in truth-value may nevertheless be either true or false, just as it may happen? This is the logical puzzle Aristotle tries to understand, and he thinks he can offer a proof that this *must* be as it is. His argument runs like this: In no case can one fact be the necessary consequence of some other fact and the opposite of this fact as well. Therefore, precisely *because* from the truth of the premisses the truth of the conclusion results by necessity, it is impossible that also from the falsity of one or both the premisses the truth of the conclusion should follow by necessity. Therefore the conclusions of syllogisms with false premisses are propositions that are by necessity not necessarily true. The same argument is applicable to the truth-or-falsity of the premisses: If the falsehood of the conclusion implies the falsity of at least one premiss, it is impossible that also from the truth of the conclusion the falsity of at least one premiss results by necessity. Therefore the conjunction of the premisses in a syllogism with true conclusion would by necessity be not necessarily false.

All this would in fact be proved if the proposition were shown to be true that one and the same fact *cannot* follow from another fact and its opposite. Now our passage 57b3–17 contains nothing but Aristotle's attempt to give a demonstration of this principle. This is not the only way to understand the text, but it is clearly a way to understand the text most thoroughly. The proof given by Aristotle is subtle, even elegant, but fallacious – as has already been pointed out by Łukasiewicz. The absurdum to which Aristotle reduces the hypothesis of the principle's being untrue looks absurd enough, but is not.

Aristotle proceeds like this: for the proposition "Both premisses are true" he substitutes the proposition "*A* is white". For the proposition "Not both the premisses are true" (the negation of the first proposition) he puts the proposition "*A* is not white". The proposition "The conclusion is true" is replaced by the proposition "*B* is large", and the proposition "The conclusion is false" by the proposition "*B* is not large".

Aristotle then gives two logical laws: (A) "If *A* implies *B*, then Not-*B* implies Not-*A*" (57b1 sqq.) and (B) "If *A* implies *B* and *B* implies *C*, then *A* implies *C*" (57b6–9). Both (A) and (B) are elementary rules of propositional logic. Now if the fact that *B* is large (the conclusion is true) could *follow* from the fact that *A* is white (both premisses are true) and *as well* from the fact that *A* is not white (not both premisses are true), we would have the two implications (1) "From '*A* is white' there follows '*B* is large'" and (2) "From '*A* is not white' there follows '*B* is large'". From (1) we get, according to (A), the implication (1′) "From '*B* is not large' there follows '*A* is not white'". Now we apply (B) to (1′) and (2) and we get the new proposition (3) "From '*B* is not large' there follows '*B* is large'" – and this is evidently absurd, says Aristotle (τοῦτο δὲ ἀδύνατον, 57b14). And Maier (l.c., II, 1, p. 331) and also Ross (l.c., p. 437) quite agree with him on this point. Both scholars think a proposition of the form "If Not-*A*, then *A*" to be self-contradictory. But, of course, only propositions of the form "Not-*A and A*" are self-contradictory, and a conjunction of two propositions is not equivalent to the conditional of these propositions, not even in Aristotle's logic. One is always tempted to think a proposition absurd when we cannot imagine a situation in which it would be the natural thing to utter this proposition. But that we cannot think of such a situation does not imply the absurdity of the proposition itself. On the other hand there may well be situations in which it is quite sensible to utter absurd propositions – examinations being one ready case of this. So the attempted proof of the principle that one fact cannot follow from another fact and the opposite of this fact breaks down here.

But it can also be shown that the principle itself is not true. There are cases which justify statements that openly violate Aristotle's principle. It is unfortunately true of some patients that they will die if operated upon and also when no operation will take place, *e.g.* in cases of advanced appendicitis complicated by extreme weakness of heart. And in such cases, apart from all general problems of causality, both a fact and its

opposite are certainly the "ground" for another fact – at least in exactly the sense Aristotle wants to rule out by his principle. For we certainly can apply Aristotle's rules (A) and (B) to our example and we will get the proposition "If the patient does not die, he will die" – which is only a rather pointless way of saying he will die anyway.

Aristotle has, to sum up, been puzzled by the logical fact that a conclusion correctly inferred from false premisses is neither by necessity true nor false. He has, to overcome his difficulty in understanding this simple fact, tried to devise a proof of it. But he failed: the main premiss of his proof (what I called his "principle") is false, and the attempted demonstration of this principle by *reductio ad absurdum* is fallacious. Now I think it is easily understood that Aristotle should feel puzzled by the logical fact that puzzled him. He freely uses rules of propositional logic in his systematic account of the syllogistic moods and figures, but he never stopped to analyse these logical structures he made use of. The truth-table-like treatment of the conditional by the Stoics somewhat later on showed that there is no puzzle here at all.

We have seen that Aristotle does not deal in our passage with the question whether syllogisms from false premisses can be valid nor with the problem whether false premisses can state the ground of the fact asserted in the conclusion. He talks simply about the relations that hold between the truth-values of premisses and conclusions in valid syllogisms, and he states them correctly. The proof he offered for the logical fact correctly stated by him was, however, shown to be incorrect. Now the interpretation of Maier is curiously different from the one just given: Maier thinks Aristotle wants to establish quite a different thesis and one that even Maier himself would not accept. But the proof Aristotle gives for this thesis Maier regards as correct. Yet it is certainly somewhat odd to reject a thesis and still to accept a proof for this thesis as correct. The interpretation given by Sir David Ross is at least self-consistent. But he, too, accepts a formally incorrect proof as valid confirmation of a thesis the proof could not establish even if it were correct.

I conclude, then, that the Albrecht argument against Łukasiewicz's thesis has no foundation in the text of the *Organon*.

NOTES

1. A.N. Prior, 'Łukasiewicz's Symbolic Logic', *Australasian Journal of Philosophy* 30 (1952), 33–46.
2. W. Albrecht, *Die Logik der Logistik*, Berlin, 1954, especially pp. 56 sqq.
3. AS, p. 49.
4. APPA, p. 436.

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